SM223 – Calculus III with Optimization Fall 2012
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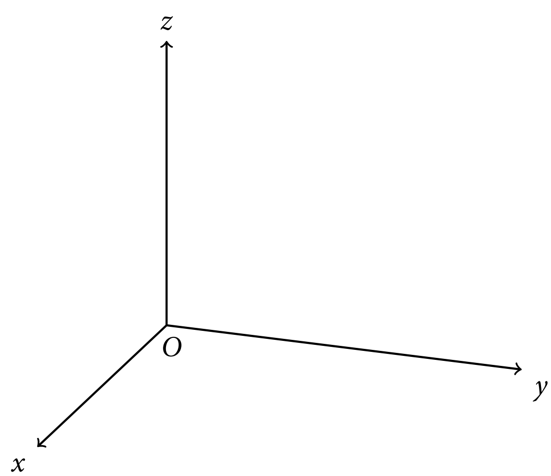
Lesson 6. Equations of Lines in 3D

1 Today...

- Vector and parametric equations
- Symmetric equations
- Vector equations for line segments
- Skew lines

2 Vector equations and parametric equations

- A **line** L is determined by a point $P_0(x_0, y_0, z_0)$ and a direction given by a vector $\vec{v}$



- The **position vector** of a point $P(a_1, a_2, a_3)$ is the vector from the origin $O(0, 0, 0)$ to the point P
- Let $\vec{r}_0$ be the position vector of P_0: that is, $\vec{r}_0 =$
- The position vector of every point on L can be expressed as the sum of $\vec{r}_0$ and a scalar multiple of $\vec{v}$
- The **vector equation** of line L is

  - Each value of the **parameter** t gives a position vector $\vec{r}$ on the line L
  - Positive values of $t \Leftrightarrow$ points on one side of P_0
  - Negative values of $t \Leftrightarrow$ points on the other side of P_0

- Let $\vec{r} = \langle x, y, z \rangle$, $\vec{v} = \langle a, b, c \rangle$
- Rewriting the vector equation component-by-component gives us the **parametric equations** of line L:

- The numbers a, b, c are called the **direction numbers** of line L
- Two lines are **parallel** if their directions are given by parallel vectors

Example 1.

(a) Find a vector equation and parametric equations for the line that passes through the point $(2, 4, 3)$ and is parallel to the vector $\vec{i} - 2\vec{j} + 4\vec{k}$.

(b) Find two other points on the line.

- The vector equation and parametric equations of a line <u>are not unique</u>
  - We can use a different point P_0
  - We can also use a different parallel vector

Example 2.

(a) Using a different point, find another set of parametric equations for the line described in Example 1.

(b) Using a different parallel vector, find another set of parametric equations for the line described in Example 1.

3 Symmetric equations

- By solving the parametric equations to eliminate t, we obtain the **symmetric equations** of line L:

Example 3.

(a) Find parametric equations and symmetric equations of the line that passes through $A(4, -3, 2)$ and $B(-1, 1, 3)$.

(b) At what point does this line intersect the xz-plane?

Example 4. Find parametric equations and symmetric equations for the line through $(2, -1, 1)$ and perpendicular to both $\langle 1, 0, 1\rangle$ and $\langle -1, 1, 0\rangle$.

4 Line segments

- Consider Example 3
- What if we just wanted to describe the **line segment** between $A(4, -3, 2)$ and $B(-1, 1, 3)$?
- Plugging in $t = 0$ to the parametric equations we found in Example 3 we get
- Plugging in $t = 1$ to the parametric equations we found in Example 3 we get

⇒ The line segment AB is described by the parametric equations

- In general, the line segment from $\vec{r}_0$ to $\vec{r}_1$ is given by the vector equation

5 Skew lines

- Two lines are **skew lines** if they do not intersect and are not parallel
  - i.e., they do not lie on the same plane

Example 5. Here are parametric equations for two lines:

$$\begin{Bmatrix} x = 1 + t \\ y = -2 + 3t \\ z = 4 - t \end{Bmatrix} \qquad \begin{Bmatrix} x = 2s \\ y = 3 + s \\ z = -3 + 4s \end{Bmatrix}$$

Show they are skew lines.