Lesson 7. Equations of Planes in 3D

1 Today...

- Skew lines
- Vector and scalar equations
  - Normal vectors

2 Skew lines

- Two lines are **parallel** if their directions are given by parallel vectors
- Two lines are **skew lines** if they do not intersect and are not parallel
  - i.e., they do not lie on the same plane

Example 1. Here are parametric equations for two lines:

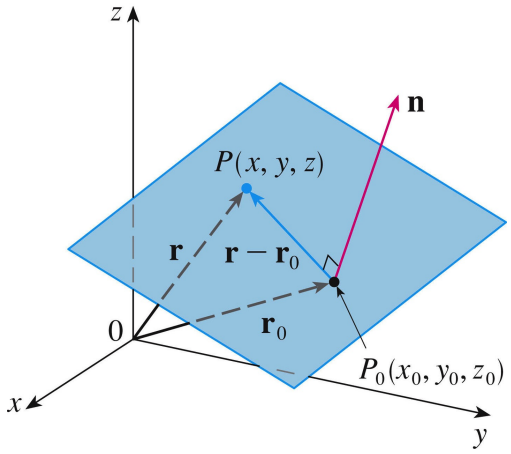
$$\left\{\begin{array}{l} x = 1 + t \\ y = -2 + 3t \\ z = 4 - t \end{array}\right\} \qquad \left\{\begin{array}{l} x = 2s \\ y = 3 + s \\ z = -3 + 4s \end{array}\right\}$$

Show they are skew lines.

3 Vector and scalar equations

- A **plane** is determined by
  - a point $P_0(x_0, y_0, z_0)$ on the plane and
  - a **normal vector** $\vec{n}$ orthogonal to the plane
- Let $\vec{r}_0$ be the position vector of P_0
- Let $\vec{r}$ be the position vector of some point on the plane

$\Rightarrow$ $\vec{r} - \vec{r}_0$ is a vector in the plane, and must be orthogonal to the normal vector $\vec{n}$



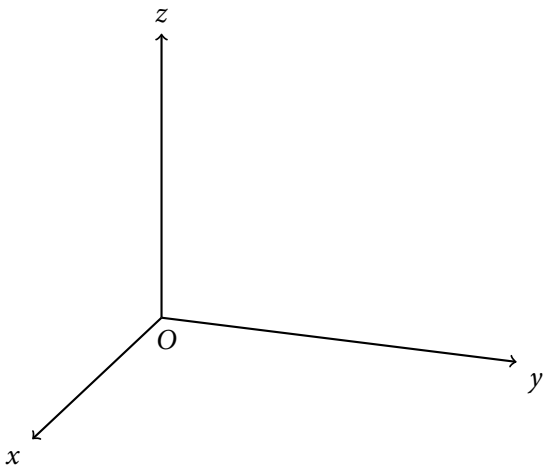
- The **vector equation** of the plane is

- Let $\vec{n} = \langle a, b, c \rangle$ and $\vec{r} = \langle x, y, z \rangle$
- Expanding the vector equation, we obtain the **scalar equation** of the plane

Example 2.

(a) Find an equation of the plane through the point $(-1, 4, 2)$ with normal vector $\vec{n} = \langle 4, 3, 2 \rangle$.

(b) Find where the plane intercepts the x-, y- and z-axes. Sketch the plane in the first orthant.



Example 3. Find an equation of the plane that passes through the points $P(1, 2, 3)$, $Q(3, 6, -1)$ and $R(5, 0, 2)$.

Example 4. Find an equation of the plane that passes through the point $(1, 2, 3)$ and contains the line $x = 3t, y = 1 + t, z = 2 - t$.