

Lesson 8. Equations of Planes in 3D, Cylinders

1 Today...

- Intersections between lines and planes
- Angles between planes and parallel planes
- Distance between a point and a plane
- Cylinders

2 Intersections between lines and planes

Example 1. Find the point at which the line

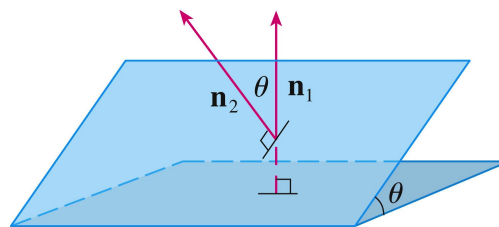
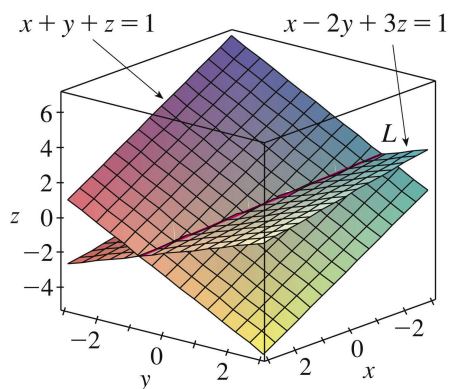
$$x = 1 + 2t, \quad y = 4t, \quad z = 2 - 3t$$

intersects the plane $x + 2y - z + 1 = 0$.



3 Angles between planes and parallel planes

- Two planes are **parallel** if their normal vectors are parallel
- If two planes are not parallel, then
 - They intersect in a straight line
 - The **angle** between the two planes is the acute angle between their normal vectors



Example 2.

- (a) Find the angle between the planes $3x - 2y + z = 1$ and $2x + y - 3z = 3$.
- (b) Find parametric equations for the line of intersection between these two planes.

4 Distance between planes

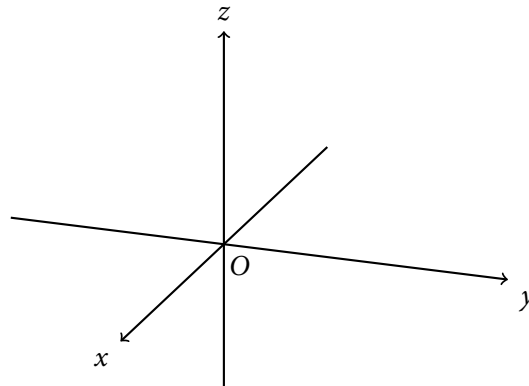
- The distance between a point $P_1(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is

Example 3. Find the distance between the parallel planes $10x + 2y - 2z = 5$ and $5x + y - z = 1$.

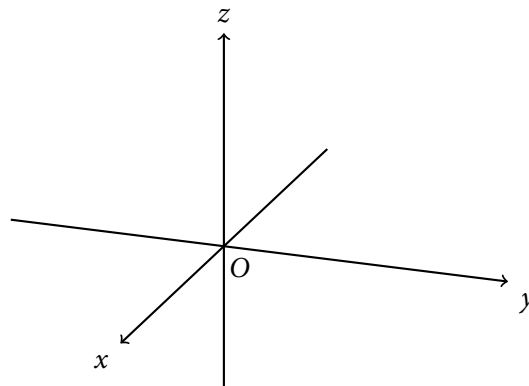
5 Cylinders

- A **cylinder** is a surface composed of all lines that
 - are parallel to a given line and
 - pass through a given plane curve
- In 3D, if one of the variables x , y , z is missing from the equation of a surface, then the surface is a cylinder.

Example 4. Sketch the graph of the surface $z = x^2$.



Example 5. Sketch the graph of the surface $y^2 + z^2 = 1$.



Example 6. Sketch the graph of the surface $xy = 1$.

