

Lesson 9. Quadric surfaces

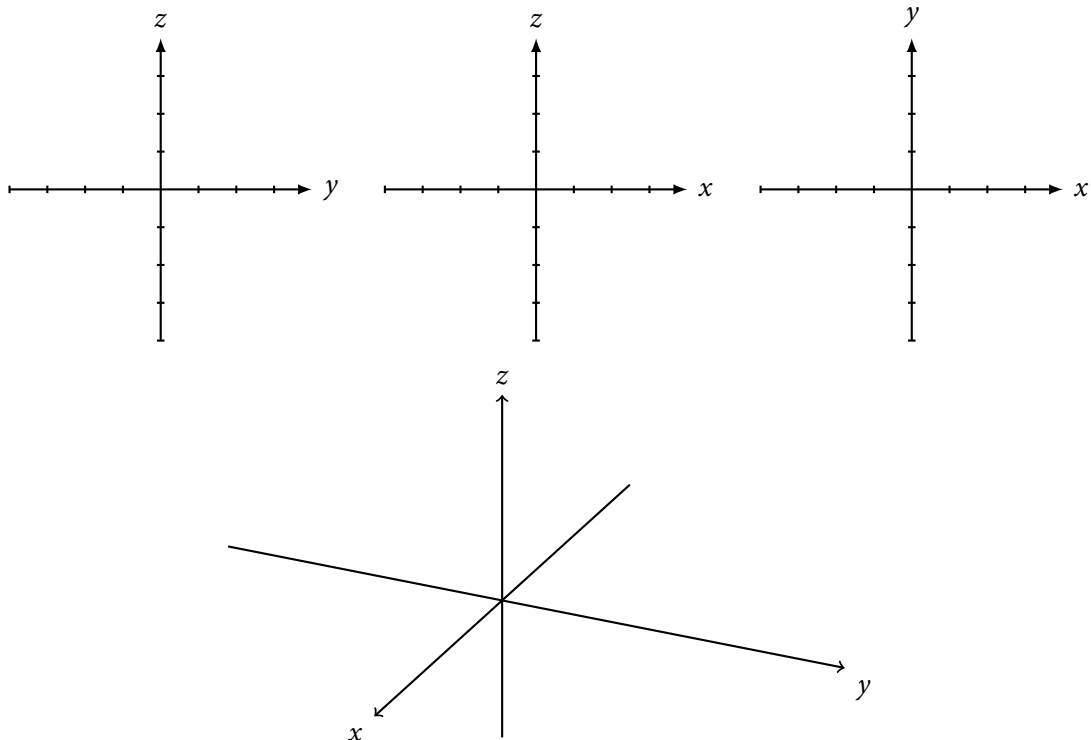
1 Today...

- Traces
- Quadric surfaces

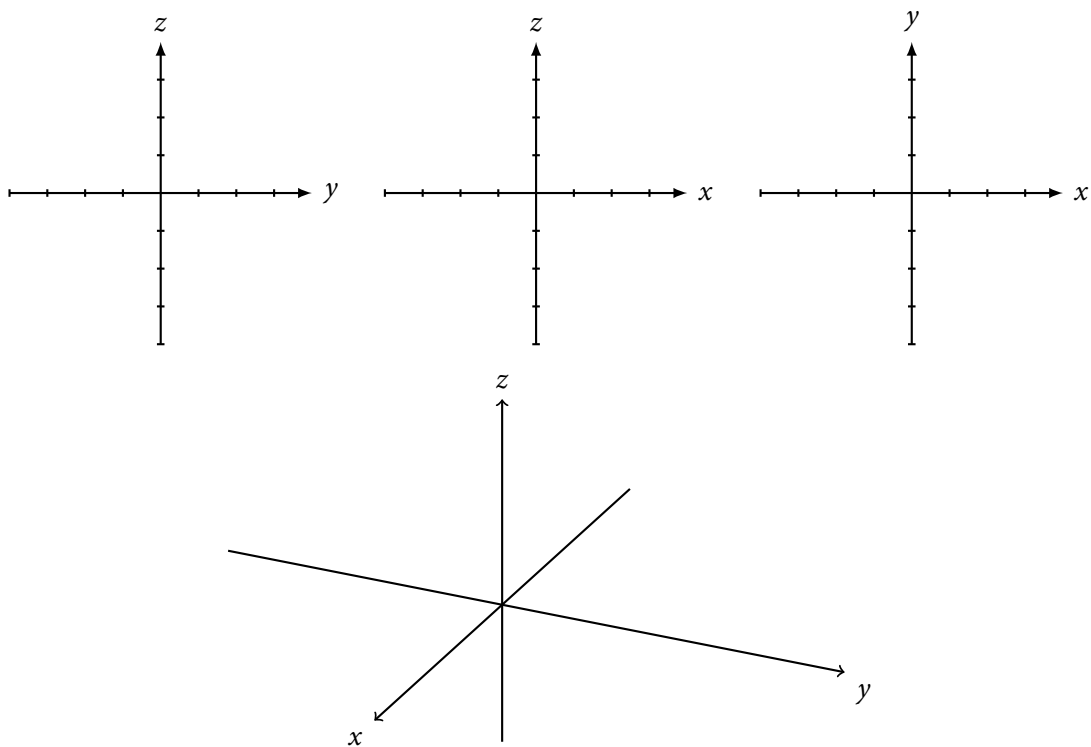
2 Traces

- A **trace** of a surface is the curve of intersection of the surface with planes parallel to the coordinate planes
- Idea:
 - Start with an equation in 3 variables x, y, z
 - Plug in a value for one of the variables
 - Graph the resulting equation in 2 variables (i.e., graph a trace of the surface)
 - Repeat for other values and other variables
 - “Glue” the traces together

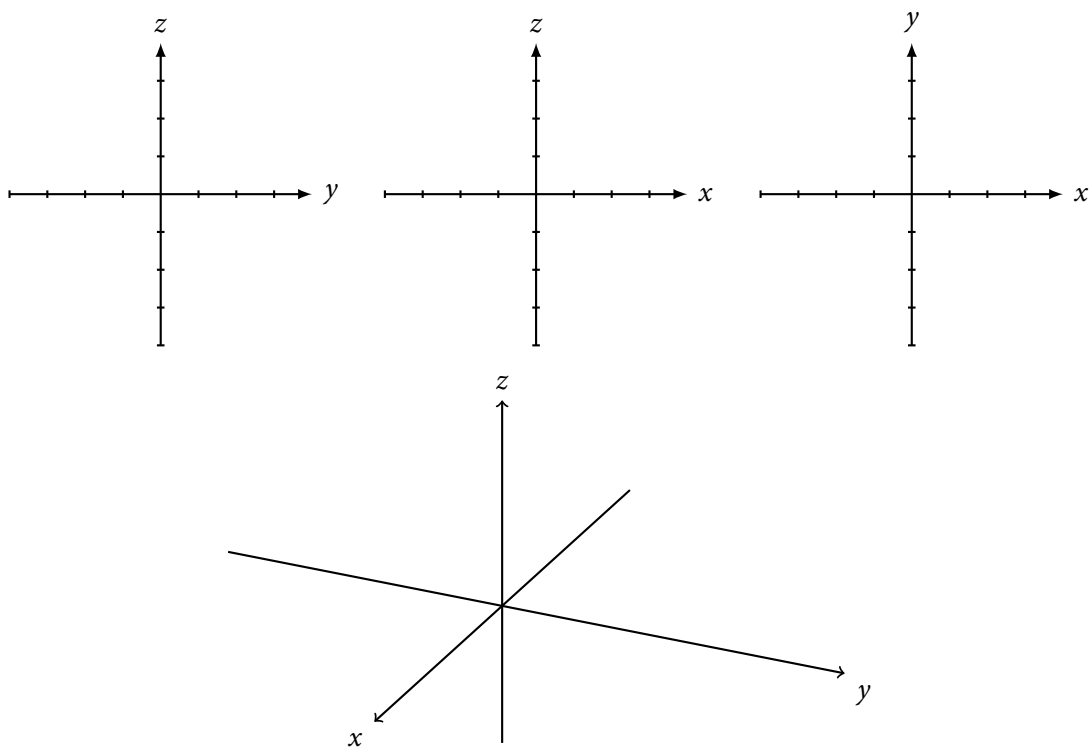
Example 1. Use traces to sketch the equation $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$.



Example 2. Use traces to sketch the surface $z = 4x^2 + y^2$.

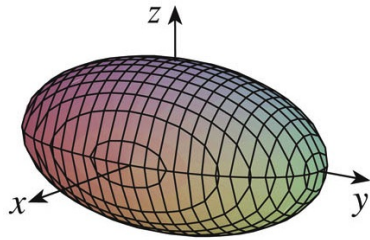


Example 3. Use traces to sketch the surface $z = y^2 - x^2$.



3 Quadric surfaces

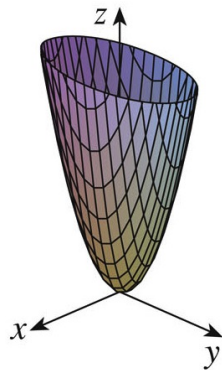
- Ellipsoid



○ Equation:

- All traces are ellipses
- If $a = b = c$, the ellipsoid is a sphere

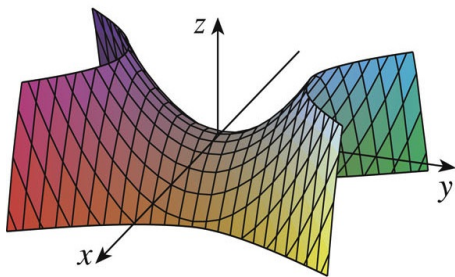
- Elliptic paraboloid



○ Equation:

- Horizontal traces are ellipses
- Vertical traces are parabolas
- The variable raised to the first power indicates the axis of the paraboloid

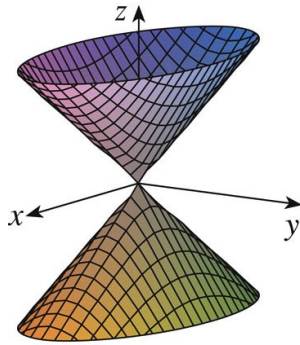
- Hyperbolic paraboloid



○ Equation:

- Horizontal traces are hyperbolas
- Vertical traces are parabolas
- The sign of c dictates the orientation of the “saddle” (the case when $c < 0$ is illustrated)

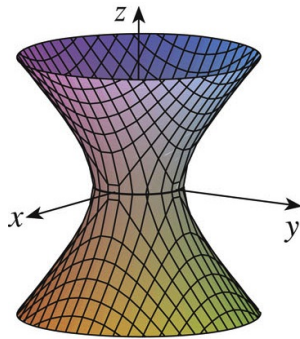
- Cone



- Equation:

- Horizontal traces are ellipses
- Vertical traces are planes or hyperbolas
- No constant indicates a cone

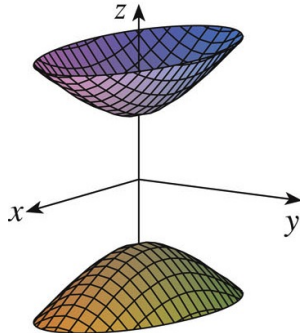
- Hyperboloid of one sheet



- Equation:

- Horizontal traces are ellipses
- Vertical traces are hyperbolas
- “-1” indicates one sheet

- Hyperboloid of two sheets



- Equation:

- Horizontal traces (when $z = k$) are ellipses if $k > c$ or $k < -c$
- Vertical traces are hyperbolas
- “+1” indicates two sheets

- Equations given above are in “standard form”

- May need to do some algebra to get an equation into standard form

- May also need to swap the variables around: for example,



is also a hyperboloid of two sheets, but with the y -axis as the central axis