SM223 – Calculus III with Optimization — Fall 2012
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Lesson 10. Vector Functions and Space Curves

1 Today...

- Vector functions
- Space curves

2 Vector functions

- A **vector function**
  - takes a real number as input and
  - outputs a vector
- For example, a 3D vector function:

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$$

  where $f(t)$, $g(t)$, and $h(t)$ are real-valued functions
  - $f(t)$, $g(t)$, and $h(t)$ are the **component functions** of $\vec{r}(t)$
- We can also have 2D vector functions: $\vec{r}(t) = \langle f(t), g(t) \rangle = f(t)\vec{i} + g(t)\vec{j}$

3 Space curves

- Suppose f, g, h are (continuous) real-valued functions
- A **space curve** is the set of all points (x, y, z) in space that satisfy

$$x = f(t) \qquad y = g(t) \qquad z = h(t)$$

  as t varies in some interval (possibly $(-\infty, +\infty)$)

$\Rightarrow$ $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ is the position vector of the point $P(f(t), g(t), h(t))$ on this curve

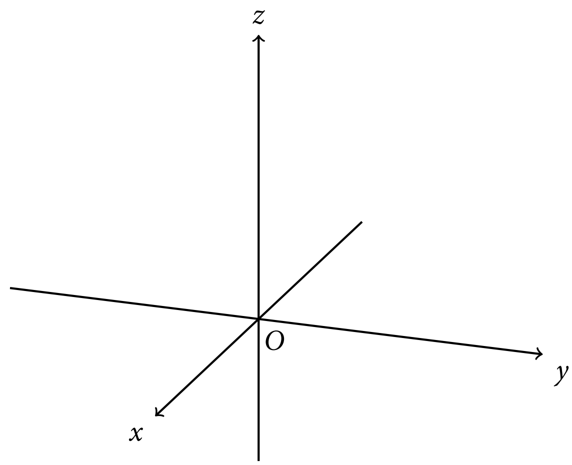
- For example, if

$$f(t) = x_0 + at \qquad g(t) = y_0 + bt \qquad h(t) = z_0 + ct$$

  for some point (x_0, y_0, z_0) and some vector $\langle a, b, c \rangle$, then $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ is a line

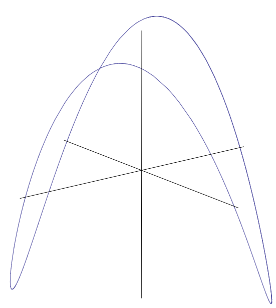
Example 1. Let $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$.

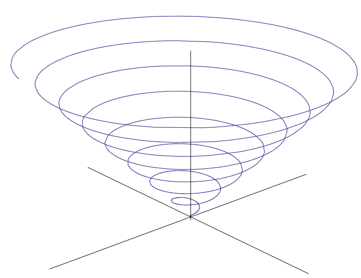
(a) Evaluate $\vec{r}(t)$ at $t = 0, \pi/2, \pi, \pi/2, 2\pi$.
(b) Sketch the curve given by $\vec{r}(t)$.

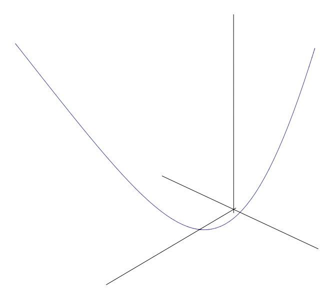


Example 2. Match the vector functions with the graphs. Give reasons for your choices.

(a) $\vec{r}(t) = \langle t\cos t, t\sin t, t \rangle$
(b) $\vec{r}(t) = \langle \cos t, \sin t, \sin 2t \rangle$
(c) $\vec{r}(t) = \langle e^{-3t/5}, t, t^2 \rangle$







Example 3. The positions of two airplanes at time t are given by the vector functions

$$\vec{r}_1(t) = \langle 1 + 2t, 1 + 6t, 1 + 14t \rangle \qquad \vec{r}_2(t) = \langle t, t^2, t^3 \rangle$$

Do the airplanes collide? *Bonus:* Do their paths intersect?