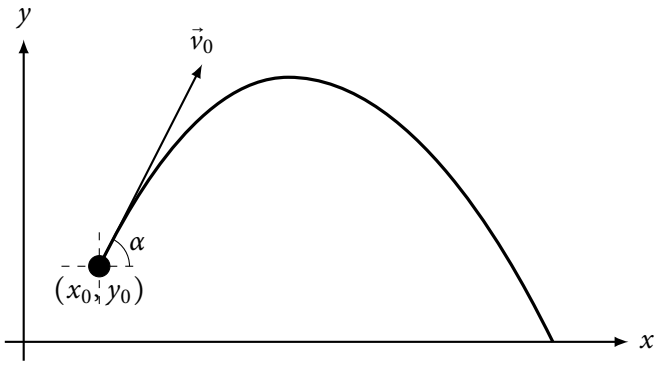
Lesson 15. Projectile Motion

1 Today...

- Trajectory of a projectile
- Horizontal distance traveled by a projectile
- Vertical height reached by a projectile, shape of the trajectory

2 Trajectory of a projectile

- A projectile with mass m is fired
  - initial point (x_0, y_0)
  - angle of elevation α
  - initial velocity $\vec{v}_0$
- Assume:
  - Air resistance is negligible
  - The only external force is due to gravity



- We (you) will derive parametric equations that describe the trajectory of this projectile
- Recall Newton's second law of motion: if at any time t, a force $F(t)$ acts on an object of mass m producing an acceleration $\vec{a}(t)$, then $\vec{F}(t) = m\vec{a}(t)$.

1. Let's define $v_0 = |\vec{v}_0|$ (we're just renaming the initial speed, or the magnitude of the initial velocity). Using this new notation, write $\vec{v}_0$ in terms of v_0 and α. *Hint.* You'll need to use trigonometry.

2. We need an expression for the acceleration $\vec{a}$ of the projectile. Since the only external force is due to gravity, which acts downward, we have that $\vec{F}(t) = m\vec{a}(t) = \langle 0, -mg \rangle$. Using this, write an expression for $\vec{a}(t)$.

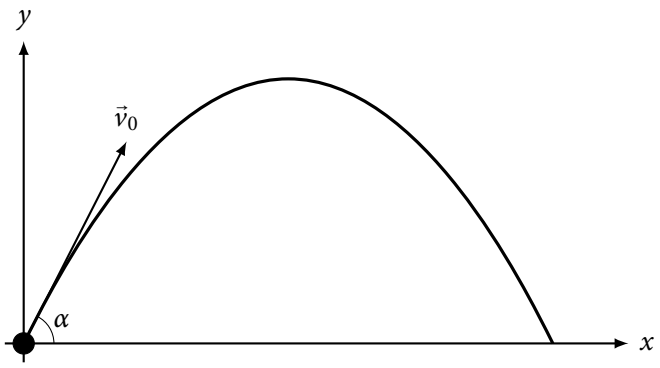
3. Using your answer from part 2, write an expression for the velocity $\vec{v}(t)$ of the projectile.
Hint 1. Recall that $\vec{a}(t) = \vec{v}'(t)$. *Hint 2.* Don't forget the constant vector of integration. *Hint 3.* Since the initial velocity is $\vec{v}_0$, we have $\vec{v}(0) = \vec{v}_0$. Use the expression for $\vec{v}_0$ you obtained in part 1.

4. Now, using your answer from part 3, write an expression for the position $\vec{r}(t)$ of the projectile.
Hint 1. Recall that $\vec{v}(t) = \vec{r}'(t)$. *Hint 2.* Don't forget the constant vector of integration. *Hint 3.* Since the initial point is (x_0, y_0), we have $\vec{r}(0) = \langle x_0, y_0 \rangle$.

5. Expand the vector equation you obtained in part 4 to write parametric equations (i.e. $x = \ldots, y = \ldots$) for the trajectory of the projectile.

3 Distance traveled by a projectile

- Let us now work under the assumption that the initial point of the projectile is $(0, 0)$: in other words, $x_0 = 0$, $y_0 = 0$.



6. The horizontal distance traveled by the projectile d is the value of x when $y = 0$. Why?

7. Set $y = 0$ and $y_0 = 0$ to your expression for y in part 5. Solve for t.

8. Use your answer in part 7 to obtain an expression for the horizontal distance traveled by the projectile. *Hint 1.* Remember that $x_0 = 0$. *Hint 2.* Use the identity $2 \sin \alpha \cos \alpha = \sin 2\alpha$.

4 Other questions

9. Again, assume that the initial point of the projectile is $(0, 0)$. What is the maximum vertical height achieved by the projectile?

10. Take your parametric equation for x in part 5 and solve for t. Plug this back into your parametric equation for y. You should have an expression for y in terms of x. This gives you an idea of how the projectile's trajectory looks like in the xy-plane. What shape does the trajectory take?