Lesson 17. Projectile Motion, continued

Example 1. David Ortiz hits a baseball at a 20° angle from 3 ft above the ground, wihch just clears the left end of the "Green Monster," the left-field wall in Fenway Park. The wall is 37 ft high and 315 ft from home plate.

(a) What was the initial speed of the ball?
(b) How long did it take the ball to reach the wall?

Note: we use $g = 32\ ft/s^2$

$x_0 = 0,\ y_0 = 3,\ \alpha = 20°$

$\Rightarrow x(t) = (v_0 \cos 20°)t,\ y(t) = 3 + (v_0 \sin 20°)t - 16t^2$

$\Rightarrow x(t) = 0.94v_0 t,\ y(t) = 3 + 0.34v_0 t - 16t^2$

Let u be the time at which the ball clears the wall

$\Rightarrow x(u) = 315,\ y(u) = 37$

$315 = 0.94v_0 u$

$\Downarrow$

$v_0 = \frac{315}{0.94u}$

$37 = 3 + 0.34v_0 u - 16u^2$

$\Rightarrow -16u^2 + 0.34\left(\frac{315}{0.94u}\right)u + 3 = 37$

$\Leftrightarrow -16u^2 + 113.94 + 3 = 37$

$\Leftrightarrow -16u^2 = -79.94$

$\Leftrightarrow u^2 = 4.99$

$\Leftrightarrow u = \pm 2.23$

$\Rightarrow$ (b) It took $\underline{\underline{2.23s}}$ for the ball to reach the wall

(a) $v_0 = \frac{315}{0.94(2.23)} = \underline{\underline{150.27\ ft/s}}$

Example 2. A projectile is fired from 100 m above the ground with an initial speed of 200 m/s and angle of elevation 60°. Find

(a) the range of the projectile,
(b) the maximum height reached, and
(c) the speed at impact.

$x_0 = 0,\ y_0 = 100,\ v_0 = 200,\ \alpha = 60^\circ$

$\Rightarrow x(t) = (200\cos 60^\circ)t = 100t$

$y(t) = 100 + (200\sin 60^\circ)t - \frac{1}{2}(9.8)t^2 = 100 + 173.21t - 4.9t^2$

(a) The projectile hits the ground when $y(t) = 0$:

$100 + 173.21t - 4.9t^2 \Rightarrow t = \dfrac{-173.21 \pm \sqrt{(173.21)^2 - 4(-4.9)(100)}}{2(-4.9)} = 35.92, -0.57$

$\Rightarrow$ The projectile lands at $t = 35.92$

Its x-position at this time (the range of the projectile) is $x(35.92) = \underline{\underline{3592}}$

(b) The maximum height occurs when $y'(t) = 0$:

$173.21 - 9.8t = 0 \Rightarrow t = \dfrac{173.21}{9.8} = 17.67$

$\Rightarrow$ The projectile's y-position at this time (the maximum height reached) is $y(17.67) = 1630.70$

(c) From (a), we know that the projectile lands at $t = 35.92$.

The velocity of the projectile at this time is

$\vec{v}(35.92) = \langle 200\cos 60^\circ,\ 200\sin 60^\circ - (9.8)(35.92) \rangle$

$= \langle 100, -178.81 \rangle$

$\Rightarrow$ The speed of the projectile at this time (time of impact) is

$|\vec{v}(35.92)| = \sqrt{100^2 + (-178.81)^2} = \underline{\underline{204.87}}$