

Lesson 18. Multivariable Functions

1 Today...

- Functions of 2 variables
- Functions of n variables
- Graphs of functions of 2 variables

2 Functions of 2 variables

- A **function f of 2 variables**
 - takes an ordered pair (x, y) of real numbers as input
 - outputs a unique real number $f(x, y)$
- The **domain D** of f is the set of allowable inputs to f
 - If f is given by a formula and its domain is not explicitly specified, then the domain of f is the set of all (x, y) for which the formula is well-defined
- The **range** of f is the set of values that f takes on

Example 1. Let $f(x, y) = \frac{\sqrt{x+y+1}}{x-1}$.

- What is $f(3, 2)$?
- What is the domain of f ?

- Example: in 1928, using economic data published by the government, Cobb and Douglas modeled production output $P(L, K)$ as a function of the amount of labor involved L and the amount of capital invested K :

$$P(L, K) = 1.01L^{0.75}K^{0.25}$$

- This function (in a more general form) is known as the *Cobb-Douglas production function*
- Note: functions are not always represented by explicit formulas
- Example: the *wind-chill index* $W(T, v)$ is a subjective temperature that is a function of the actual temperature T (in $^{\circ}\text{C}$) and wind speed v (in km/h), as given by the table below:

		Wind speed (km/h)											
		v	5	10	15	20	25	30	40	50	60	70	80
Actual temperature ($^{\circ}\text{C}$)	T												
	5		4	3	2	1	1	0	-1	-1	-2	-2	-3
	0		-2	-3	-4	-5	-6	-6	-7	-8	-9	-9	-10
	-5		-7	-9	-11	-12	-12	-13	-14	-15	-16	-16	-17
	-10		-13	-15	-17	-18	-19	-20	-21	-22	-23	-23	-24
	-15		-19	-21	-23	-24	-25	-26	-27	-29	-30	-30	-31
	-20		-24	-27	-29	-30	-32	-33	-34	-35	-36	-37	-38
	-25		-30	-33	-35	-37	-38	-39	-41	-42	-43	-44	-45
	-30		-36	-39	-41	-43	-44	-46	-48	-49	-50	-51	-52
	-35		-41	-45	-48	-49	-51	-52	-54	-56	-57	-58	-60
	-40		-47	-51	-54	-56	-57	-59	-61	-63	-64	-65	-67

- If the temperature is -15°C and the wind speed is 40 km/h, then the wind-chill index is

Example 2. Give an example of a function of 2 variables that models some real-world phenomenon. Be creative! In particular:

- Explain what the variables (inputs) represent.
- Explain what the function value (output) represents.
- Give a formula for your function (make up something somewhat reasonable).

3 Functions of n variables

- A function f of n variables

- takes an ordered tuple (x_1, \dots, x_n) of real numbers as input
- outputs a unique real number $f(x_1, \dots, x_n)$

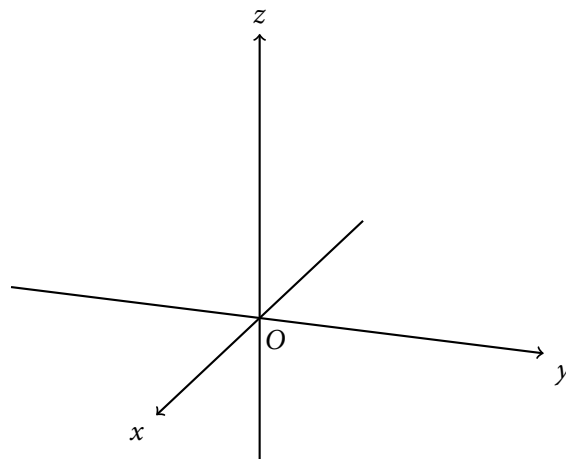
- Example:

- Anteater-Bugs produces n types of beers
- It costs c_i to produce one bottle of beer i , $i = 1, \dots, n$
- Say that x_i bottles of beer i are produced, $i = 1, \dots, n$
- Cost of producing x_1 bottles of beer 1, x_2 bottles of beer 2, etc.:

4 Graphs of functions in 2 variables

- Let f be a function of 2 variables with domain D
- The **graph** of f is the set of all points (x, y, z) in \mathbb{R}^3 such that $z = f(x, y)$ and (x, y) is in D

Example 3. Sketch the graph of $f(x, y) = x^2 + 2y^2$.



- If the function defines a plane or quadric surface, then figuring out the graph is pretty easy
- What about more complicated functions? For example:
 - The Cobb-Douglas function: $f(x, y) = 1.01x^{0.75}y^{0.25}$
 - $f(x, y) = \sin x + \sin y$
 - $f(x, y) = \frac{x^2 + 3y^2}{e^{-x^2+y^2}}$