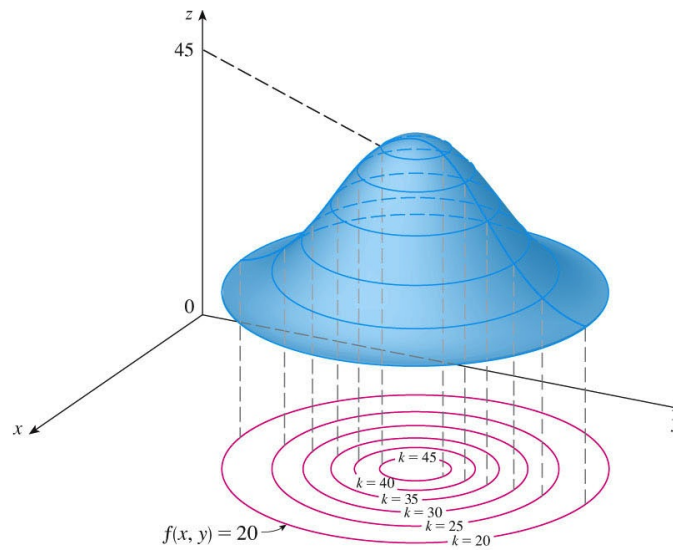


Lesson 19. Level Curves

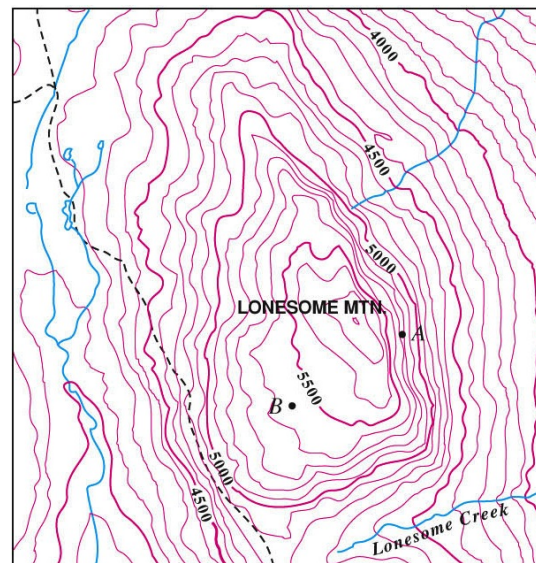
- Another way of visualizing functions of 2 variables: level curves
- The **level curves** of a function $f(x, y)$ are the curves of the equations ,
where k is a constant

- These curves show where the graph of f has height k , for different values of k
- Sometimes called *contour maps*

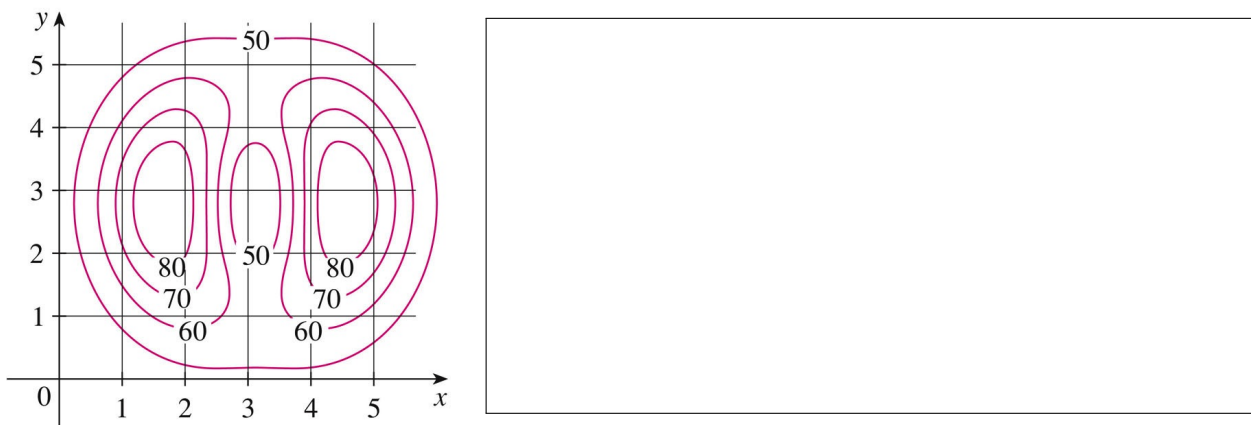


- Level curves are close together \Leftrightarrow surface is steep
- Level curves are farther apart \Leftrightarrow surface is flatter

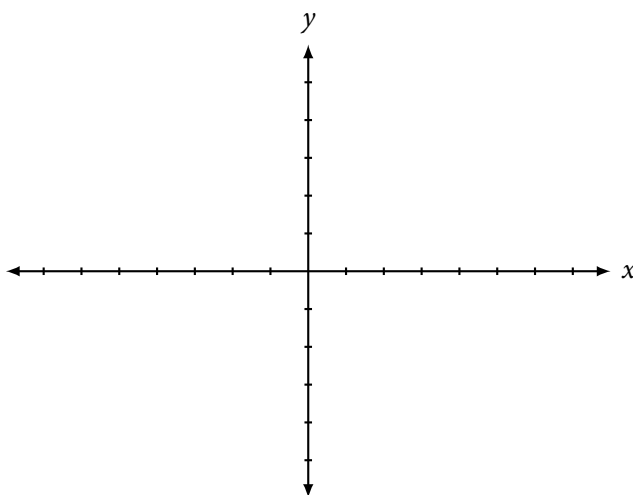
- Example: topographic maps
- Contour lines are curves of



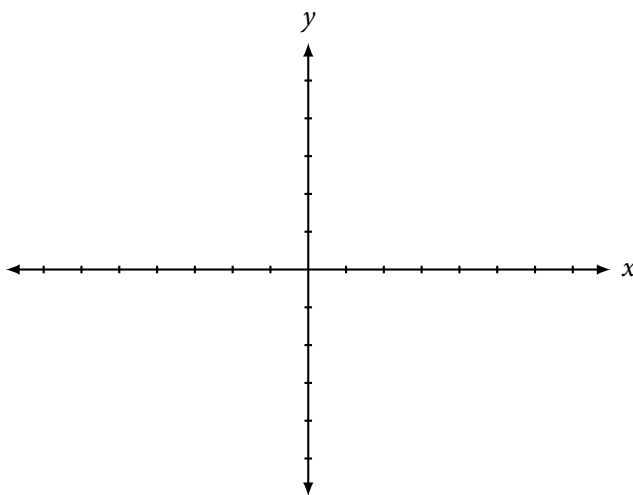
Example 1. The contour map for a function f is given below. Use it to estimate the values of $f(1, 3)$ and $f(4, 5)$.



Example 2. Sketch the level curves of the function $f(x, y) = 6 - 3x - 2y$ for the values $k = -6, 0, 6, 12$.



Example 3. Sketch the level curves of the function $f(x, y) = \sqrt{x} - y$ for the values $k = 0, 1, 2, 3$.



- What about those crazy functions from last time?
 - The Cobb-Douglas function: $f(x, y) = 1.01x^{0.75}y^{0.25}$
 - $f(x, y) = \sin x + \sin y$
 - $f(x, y) = \frac{x^2 + 3y^2}{e^{-x^2+y^2}}$

Example 4. Match the function with its graph and contour map.

- (a) $f(x, y) = (1 - x^2)(1 - y^2)$
 (b) $f(x, y) = \sin(x - y)$
 (c) $f(x, y) = e^x \cos y$

