Lesson 21.

Problems 1-5: See solutions on WebAssign or course website.

Problem 6. $f(x,y,z) = \sqrt{\sin^2 x + \sin^2 y + \sin^2 z}$

$$\Rightarrow f_x(x,y,z) = \frac{1}{2}\left(\sin^2 x + \sin^2 y + \sin^2 z\right)^{-\frac{1}{2}}\left(2\sin x \cos x\right) = \frac{\sin x \cos x}{\sqrt{\sin^2 x + \sin^2 y + \sin^2 z}}$$

$$\Rightarrow f_x\left(0,0,\tfrac{\pi}{4}\right) = 0.$$

Problem 7. $f(x,y) = \cos(x^2 y)$

$$\Rightarrow f_x(x,y) = -\sin(x^2y)(2xy) = -2xy\sin(x^2y)$$

$$f_y(x,y) = -\sin(x^2y)(x^2) = -x^2\sin(x^2y)$$

$$\Rightarrow f_{xy}(x,y) = (-2xy)(\cos(x^2y))(x^2) + (-2x)\sin(x^2y)$$
$$= -2x^3y\cos(x^2y) - 2x\sin(x^2y)$$

$$f_{yx}(x,y) = (-x^2)(\cos(x^2y))(2xy) + (-2x)\sin(x^2y)$$
$$= -2x^3y\cos(x^2y) - 2x\sin(x^2y)$$

$\Rightarrow f_{xy} = f_{yx}$

Problem 8. $f(x,y) = x^4y^2 - x^3y$

$f_x(x,y) = 4x^3y^2 - 3x^2y$

$f_{xx}(x,y) = 12x^2y^2 - 6xy$

$f_{xxx}(x,y) = 24xy^2 - 6y$

$f_{xy}(x,y) = 8x^3y - 3x^2$

$f_{xyx}(x,y) = 24x^2y - 6x$