

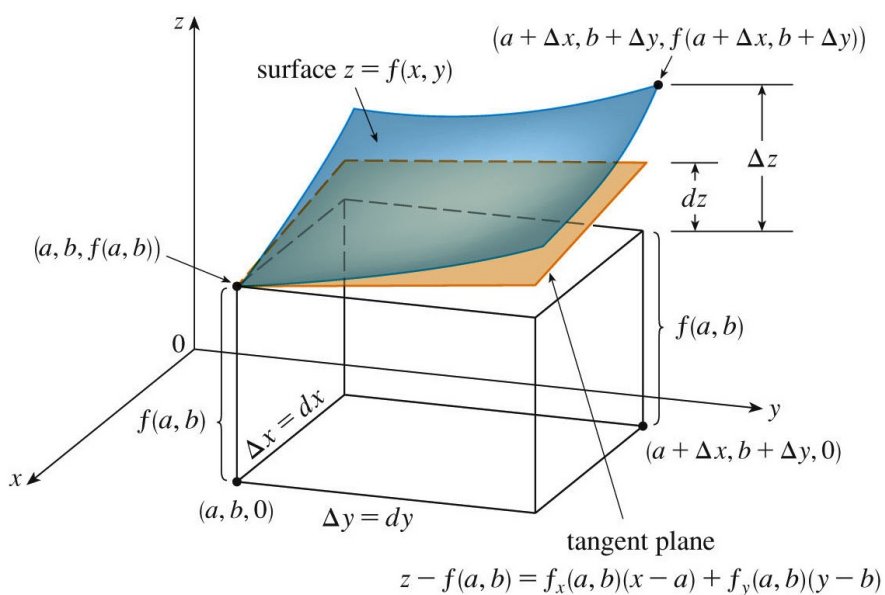
Lesson 23. Differentials

- Suppose we want to find the difference between $f(x, y)$ and $f(a, b)$: $f(x, y) - f(a, b)$
- Recall that the linear approximation of f at (a, b) is

- Letting $dx = x - a$ and $dy = y - b$ and rearranging terms, we get

- The **differential** of f is:

- So we can approximate the difference between $f(x, y)$ and $f(a, b)$ by computing the differential df



- A similar idea holds for functions of three or more variables: e.g., for $f(x, y, z)$, the differential is:

Example 1. Let $f(x, y) = x^2 + 3xy - y^2$.

- (a) Find the differential df .
- (b) Use the differential to approximate the change in f when x changes from 2 to 2.05 and y changes from 3 to 2.96.
- (c) Compare your approximation with the actual change.

Example 2. The height and base radius of a cylinder are measured as 10 cm and 2 cm, respectively, with a measurement error of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated volume of the cylinder.

Example 3. Three positive numbers, each less than 100, are rounded to the first decimal place and then multiplied together. Use differentials to estimate the maximum possible error in the computed product that might result from rounding.