

Lesson 24. The Chain Rule

1 Today

- Two special cases of the chain rule
- Tree diagrams and the general version of the chain rule

2 Case 1

- Let $z = f(x, y)$ be a function of 2 variables
- Let x and y be functions of 1 variable: $x = g(t)$ and $y = h(t)$

  $\Rightarrow$ z is indirectly a function of t: $z = f(g(t), h(t))$
- Can we find the derivative of z with respect to t?
- **Chain rule (Case 1)**:

Example 1. Let $z = x^2 y + 3xy^4$, $x = \sin 2t$, and $y = \cos t$.

(a) Find dz/dt.
(b) Find dz/dt when $t = 0$.

3 Case 2

- Let $z = f(x, y)$ be a function of 2 variables
- Let x and y be functions of 2 variables: $x = g(s, t)$ and $y = h(s, t)$

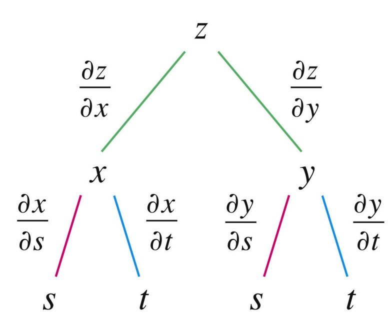
  $\Rightarrow$ z is indirectly a function of s and t: $z = f(g(s, t), h(s, t))$
- We can find the derivative of z with respect to s and t

- **Chain rule (Case 2)**:

Example 2. Let $z = \sin x \cos y$, $x = st^2$, $y = s^2 t$. Find $\partial z/\partial s$ and $\partial z/\partial t$.

4 Tree diagrams

- How can we remember the chain rule (Case 2)?
- Draw a **tree diagram**:



- To get $\partial z/\partial s$, follow all the paths from z to s:

- This idea can be extended in general to functions of 3 or more variables

Example 3. Write out the chain rule for the case where $z = f(w, x, y)$, $w = g(s, t)$, $x = h(s, t)$, $y = \ell(s, t)$.

Example 4. The length ℓ, width w, and height h of a box change with time. At a certain instant the dimensions are $\ell = 1$ m, $w = 2$ m, and $h = 2$ m. ℓ and w are increasing at a rate of 2 m/s while h is decreasing at a rate of 3 m/s. Find the rate at which the length of the diagonal is changing at that instant.