SM223 – Calculus III with Optimization
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Lesson 27. The Gradient Vector and Directional Derivatives

0 Warm up

Example 1. Let $\vec{a} = 4\vec{i} + \vec{j}$ and $\vec{b} = \vec{i} - 2\vec{j}$.

(a) Find $\vec{a} \cdot \vec{b}$.
(b) Find a unit vector that has the same direction as $\vec{b}$.

1 The gradient vector

- The **gradient** of a function $f(x, y)$ of two variables is

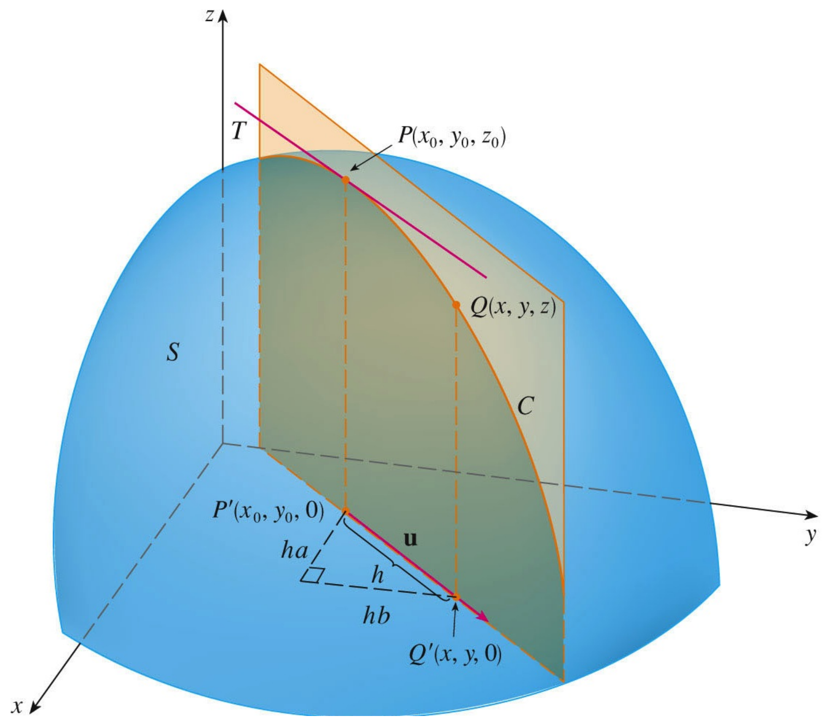
- The gradient is a vector of partial derivatives

Example 2. Let $f(x, y) = \sin y + e^{xy}$. Find (a) $\nabla f(x, y)$, (b) $\nabla f(1, 0)$.

2 The directional derivative

- Recall for a function $f(x, y)$:
  - The partial derivative f_x is
  - The partial derivative f_y is
- What about other directions?

- Let $u = \langle a, b \rangle$ be an arbitrary unit vector

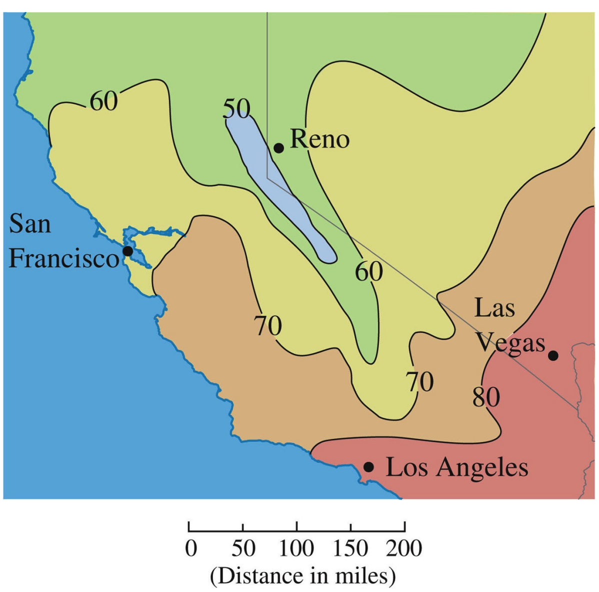


- The **directional derivative** of f at (x, y) in the direction of a unit vector $\vec{u} = \langle a, b \rangle$ is

$$D_{\vec{u}} f(x, y) = \lim_{h \to 0} \frac{f(x + ha, y + hb) - f(x, y)}{h}$$

- The directional derivative $D_{\vec{u}} f(x, y)$ is

Example 3. The contour map of the temperature function $T(x, y)$ is shown below (x and y are simply coordinates). Estimate the directional derivative of T at Reno in the southeasterly direction. What does this value mean?



- To compute the directional derivative, we can use:

- Note: $\vec{u}$ must be a unit vector
  - If you are asked for the the directional derivative "in the direction of $\vec{v}$," make sure $\vec{v}$ is a unit vector. If it isn't, make it one.

Example 4. Find the directional derivative of $f(x, y) = \sin y + e^{xy}$ at the point $(1, 0)$ in the direction of the vector $\vec{v} = \langle -3, 4 \rangle$.

3 The gradient and directional derivative for functions of 3 variables

- The gradient of a function $f(x, y, z)$ of three variables is defined similarly:

$$\nabla f(x, y) = \langle f_x(x, y), f_y(x, y), f_z(x, y) \rangle$$

- The directional derivative of f at (x, y, z) in the direction of a unit vector $\vec{u}$ can be computed using

$$D_{\vec{u}} f(x, y, z) = \nabla f(x, y, z) \cdot \vec{u}$$

- The directional derivative $D_{\vec{u}} f(x, y, z)$ is

Example 5. Find the directional derivative of $f(x, y, z) = \ln(3x + 6y + 9z)$ at point $(1, 1, 1)$ in the direction of $\vec{v} = \langle 2, 6, 3 \rangle$.