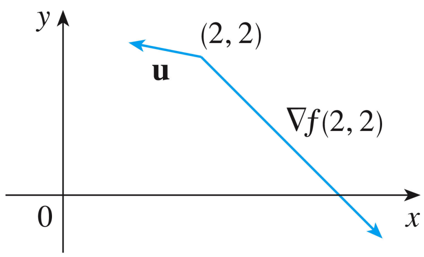
SM223 – Calculus III with Optimization Fall 2012
Asst. Prof. Nelson Uhan

# Lesson 28. Maximizing the Directional Derivative

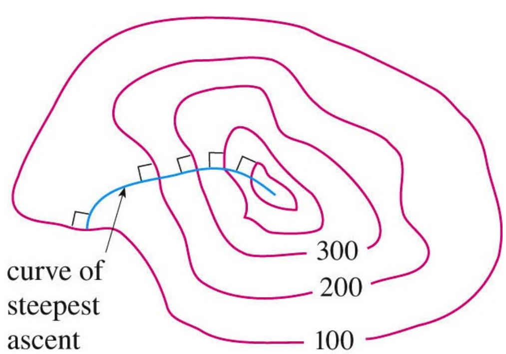
## 0 Warm up

**Example 1.** Use the figure below to estimate $D_{\vec{u}} f(2,2)$.




## 1 Maximizing the directional derivative

- From last time: in words, the directional derivative of $f$ at $(x, y)$ in the direction of unit vector $\vec{u}$ is
- Questions:
  - In which direction does $f$ change the fastest? (steepest ascent or descent)
  - What is this maximum rate of change?
- **Important theorem:** ($f$ is a function of 2 or 3 variables)
  - The maximum value of $D_{\vec{u}} f$ is $|\nabla f|$
  - The maximum value occurs when $\vec{u}$ is in the same direction as $\nabla f$




- As a result, the gradient is

**Example 2.** Let $f(x, y) = xe^y$.

(a) Find the rate of change of $f$ at the point $P(2, 0)$ in the direction from $P$ to $Q(\frac{1}{2}, 2)$.

(b) In what direction does $f$ have the maximum rate of change? What is this maximum rate of change?

## 2 On your own

**Example 3.** Find the directional derivative of $f(x, y) = \sqrt{xy}$ at $P(2, 8)$ in the direction of $Q(5, 4)$.

**Example 4.** Let $f(x, y) = \sqrt{x^2 + y^2 + z^2}$. Find the maximum rate of change of $f$ at $(3, 6, -2)$ and the direction in which it occurs.

**Example 5.** Find all points at which the direction of fastest change of the function $f(x, y) = x^2 + y^2 - 2x - 4y$ is $\vec{i} + \vec{j}$. *Hint.* Your answer should be an equation in $x$ and $y$.