Lesson 30. Introduction to Operations Research and Optimization

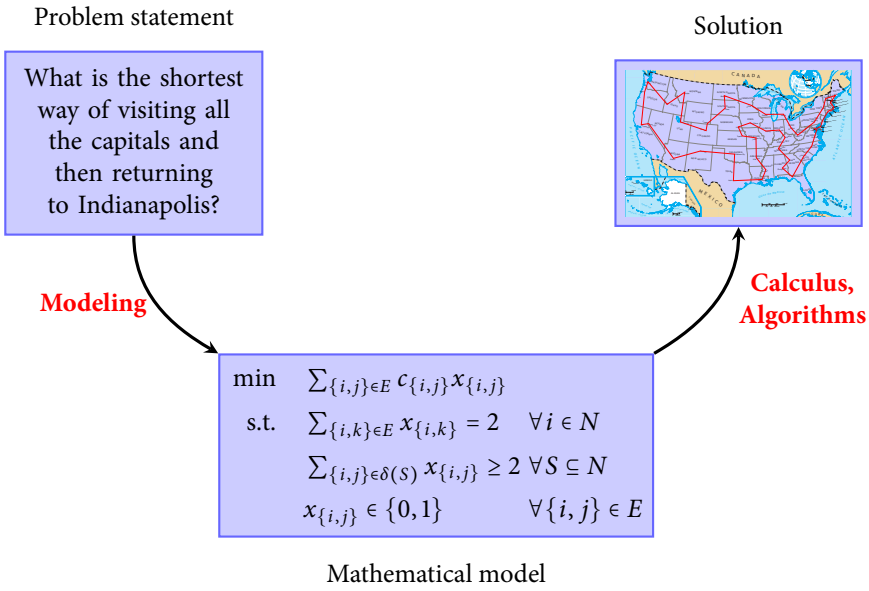
What is operations research?

- "The most influential academic discipline field you've never heard of" [Boston Globe, 2004]
- **Operations Research (OR)** is the discipline of applying advanced mathematical methods to help make better decisions
- "The Science of Better" [INFORMS slogan]
- "A liberal education in a technological world" [Thomas Magnanti, former Dean of Engineering at MIT]
- Numerous applications
  - military logistics in World War II
  - manufacturing
  - operating systems
  - logistics
  - airline pricing
  - communications
  - finance
  - marketing
- We'll talk about some applications later

The traveling salesperson problem

- A saleswoman located in Annapolis wants to visit all 48 state capitals of the continental United States to sell her wares
- What is shortest way of visiting all the capitals and then returning to Annapolis?
- Entire books have been written on the TSP
- 1962: contest by Proctor and Gamble - best TSP tour through 33 US cities
- 1998: The Florida Sun-Sentinel's Science page ponders Santa Claus's traveling problem
- One of the most popular problems in operations research
- Numerous applications in expected and unexpected places
  - Circuit board manufacturing
  - Genome sequencing
- Your turn! Try to find the shortest way of visiting all the capitals and then returning to Indianapolis
- What about 13,509 cities in the US?
- Sophisticated mathematical techniques are our best bet

The OR approach



Optimization is everywhere

- "Minimize" time it takes to get from class to class
- "Maximize" the company's profits
- (Moneyball) "Best" lineup for the Oakland A's
- We are always trying to make **decisions** in a way that meets some **objective** subject to some **constraints**
- Some success stories of optimization helping solve complex real-world decision-making problems ...

Sports scheduling

- ACC Basketball earns over $30 million in revenue annually, almost all from TV and radio
- TV networks need a steady stream of "high quality" games, NCAA rules, school preferences and traditions
- **Decision:**

- **Objective:**

- **Constraints:**

- Optimization approaches yields reasonable schedules very quickly

Radiation therapy

- High doses of radiation can kill cancer cells and/or prevent them from growing and dividing
- Can also kill healthy cells!
- Radiation can be delivered at different angles and intensities
- **Decision:**

- **Objective:**

- **Constraints:**

- Many successes reported using different types of optimization models

My first optimization model

- Let's try to build a simple optimization model

The Anteater-Bugs Corporation [Adapted from MIT 15.053, Spring 2007]

- The Anteater-Bugs Corporation has two main brands of beer: Bugwheezer and Bug-Lite
- Each of these products contains two main ingredients: malt and hops
- Beer production also requires some labor
- Ann Anteater is in charge of beer production for October

- For October:

Ingredient / Beer	Bugwheezer	Bug-Lite	Total available
Malt (grams/bottle)	10	3	1000
Hops (grams/bottle)	20	12	2000
Profit ($/bottle)	1	2	

What goes into an optimization model?

1. Determine the **decision variables**
2. Write the **objective function** (and goal) in terms of the decision variables
3. Write the **constraints** in terms of the decision variables

- Get something that looks like this:

$$\begin{aligned} \text{minimize or maximize} \quad & \text{(objective function)} \\ \text{subject to} \quad & \text{(constraints)} \end{aligned}$$

Step 1: Determine the decision variables

- The **decision variables** in an optimization model represent the decisions to be taken, for example
  - "How much of product A should be produced?"
- Decision variables should completely describe the decisions to be made
- Ann needs to determine the Anteater-Bugs's October production quantities of Bugwheezer and Bug-Lite
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Step 2: Write the objective function

- The **objective function** of an optimization model quantifies the quality of the decisions described by the decision variables, for example
  - total cost of producing products A, B and C
- The objective function can be **maximized** or **minimized** (the **goal**)
- The Anteater-Bugs Corporation wants to maximize profits

- Assume all bottles produced are sold
- If W bottles of Bugwheezer and L bottles of Bug-Lite are produced, what is the profit?
- Objective function + goal:

Step 3: Write the constraints

- **Constraints** specify the the values that decision variables can take through equalities and inequalities
- If W bottles of Bugwheezer and L bottles of Bug-Lite are produced, how much malt is used?
- This quantity needs to be less than 1000
- This constraint can be written as
- Similar constraint for hops:
- Can we produce a negative number of beer bottles?
- Can we produce a fractional number of beer bottles?
- Nonnegativity:
- Integrality:

Anteater-Bugs's Optimization Model

- Putting this all together...

$$
\begin{array}{lrl}
\text{maximize} & 1W + 2L & \text{(total profit)} \\
\text{subject to} & 10W + 3L \leq 1000 & \text{(malt capacity)} \\
& 20W + 12L \leq 2000 & \text{(hops capacity)} \\
& W \geq 0 & \text{(nonnegativity)} \\
& L \geq 0 & \\
& W \text{ integer} & \text{(integrality)} \\
& L \text{ integer} &
\end{array}
$$

Goals for this course

- Formulate very simple models
- Use calculus to solve even simpler models

Next time...

- The calculus of optimization