

## Lesson 31. Local Minima and Maxima

### 0 Review

**Example 1.** The temperature at a point  $(x, y, z)$  is given by

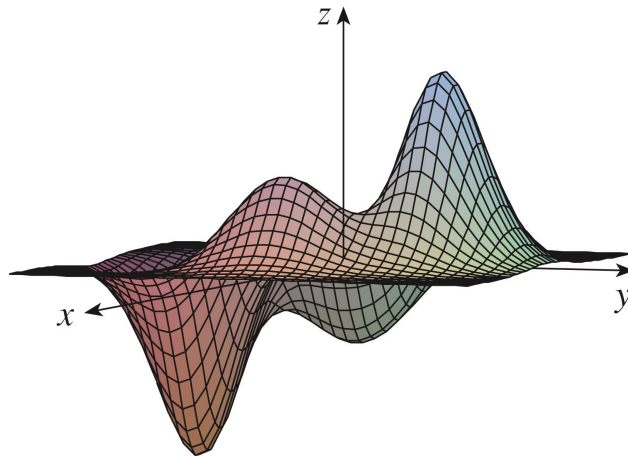
$$T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$$

where  $T$  is measured in  $^{\circ}\text{C}$  and  $x, y, z$  in meters.

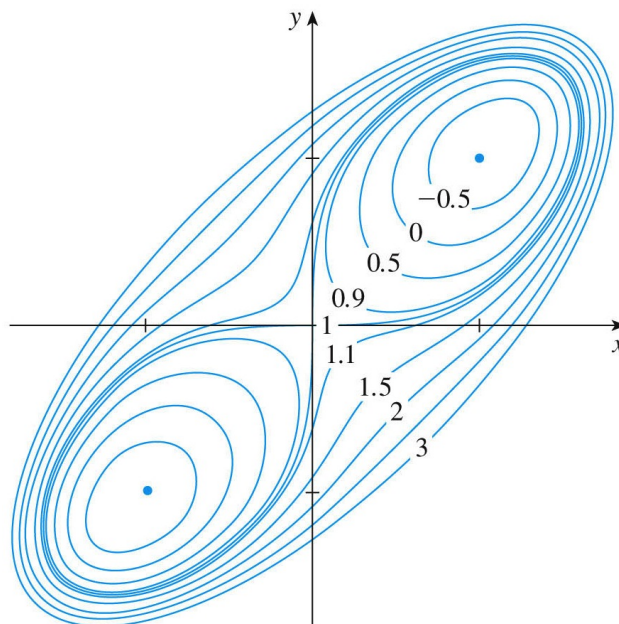
- Find the rate of change of temperature at the point  $P(2, -1, 2)$  in the direction toward the point  $(3, -3, 3)$ .
- In which direction does the temperature increase fastest at  $P$ ?
- Find the maximum rate of increase at  $P$ .

## 1 Local minima and maxima

- Let  $f$  be a function of two variables
- $(a, b)$  is a **local maximum** of  $f$  if  $f(a, b) \geq f(x, y)$  for all  $(x, y)$  “close” to  $(a, b)$ 
  - $f(a, b)$  is a **local maximum value**
- $f$  has a **local minimum** at  $(a, b)$  if  $f(a, b) \leq f(x, y)$  for all  $(x, y)$  “close” to  $(a, b)$ 
  - $f(a, b)$  is a **local minimum value**



**Example 2.** The contour map for  $f(x, y) = x^4 + y^4 - 4xy + 1$  is shown below. Find the local maxima and minima of  $f$ .



## 2 Critical points: how to find local minima and maxima

- $(a, b)$  is a **critical point** of  $f$  if

or if one of these partial derivatives does not exist

- If  $(a, b)$  is a local minimum or maximum, then  $(a, b)$  is a critical point
- Finding local minima/maxima of  $f$ :

1. Find all critical points of  $f$

2. Categorize each critical point using the **second derivatives test**:

- Let  $D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$

- If  $D > 0$  and  $f_{xx}(a, b) > 0$ , then  $(a, b)$  is a  of  $f$

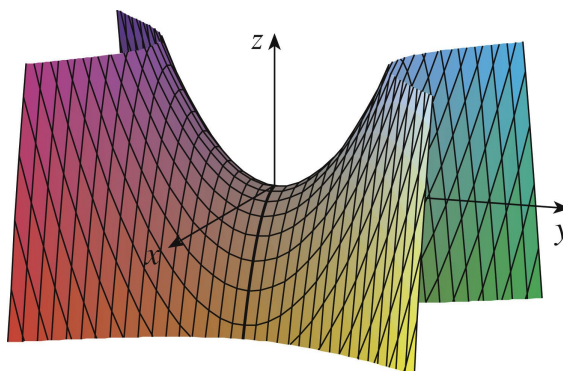
- If  $D > 0$  and  $f_{xx}(a, b) < 0$ , then  $(a, b)$  is a  of  $f$

- If  $D < 0$ , then  $(a, b)$  is a  of  $f$

- If  $D = 0$ , the test gives no information

- Saddle points

- Highest point in one direction, lowest point in the other direction
- Graphically:



- Saddle points look like hyperbolas in contour maps (see  $(0, 0)$  in Example 2)

- Solving systems of equations with the TI-nspire CX:

- Press Menu
- Select Algebra
- Select Solve System of Equations
- Select Solve System of Equations...

**Example 3.** Find the local minima and maxima and saddle points of  $f(x, y) = x^4 + y^4 - 4xy + 1$ .

