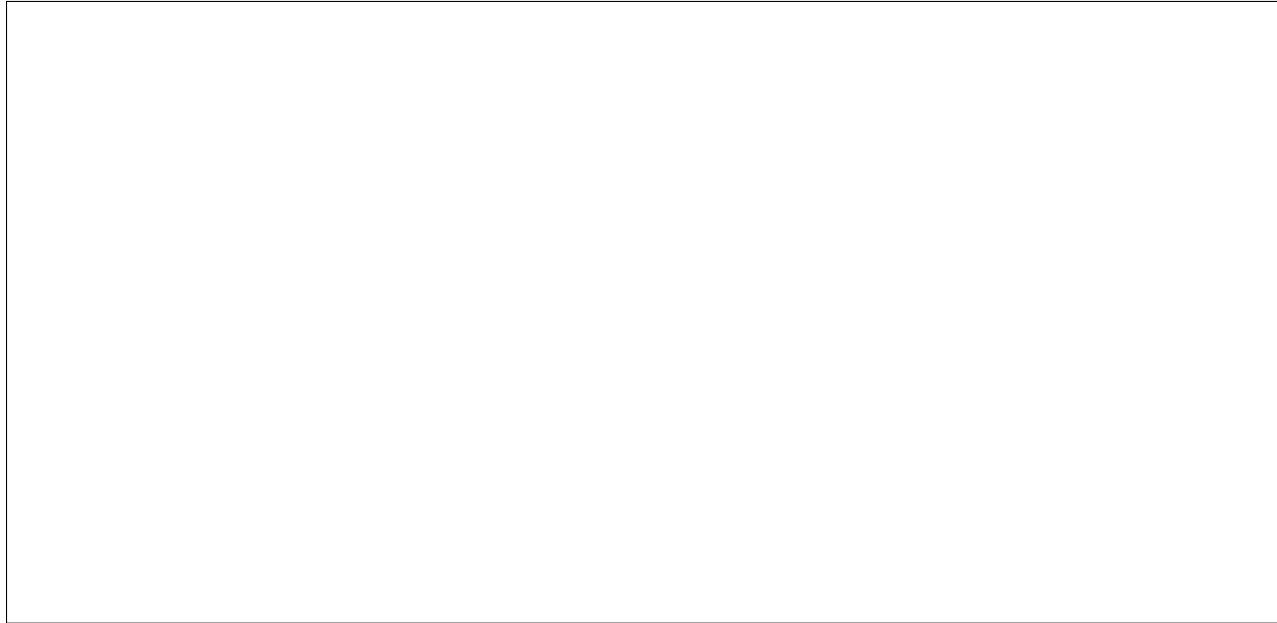


## Lesson 33. Constrained Optimization

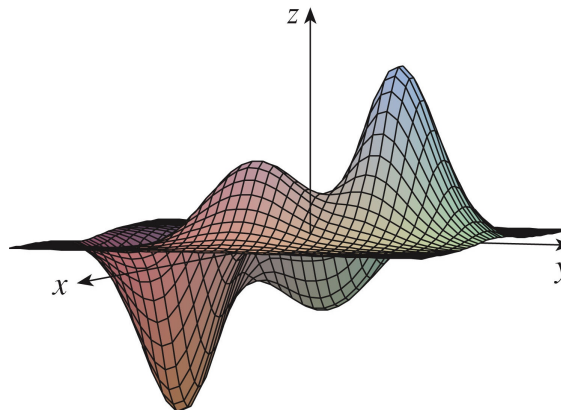
### 0 Review

**Example 1.** Find the local maxima and minima and saddle points of  $f(x, y) = (x^2 + y)e^{y/2}$ .



### 1 Absolute minima and maxima

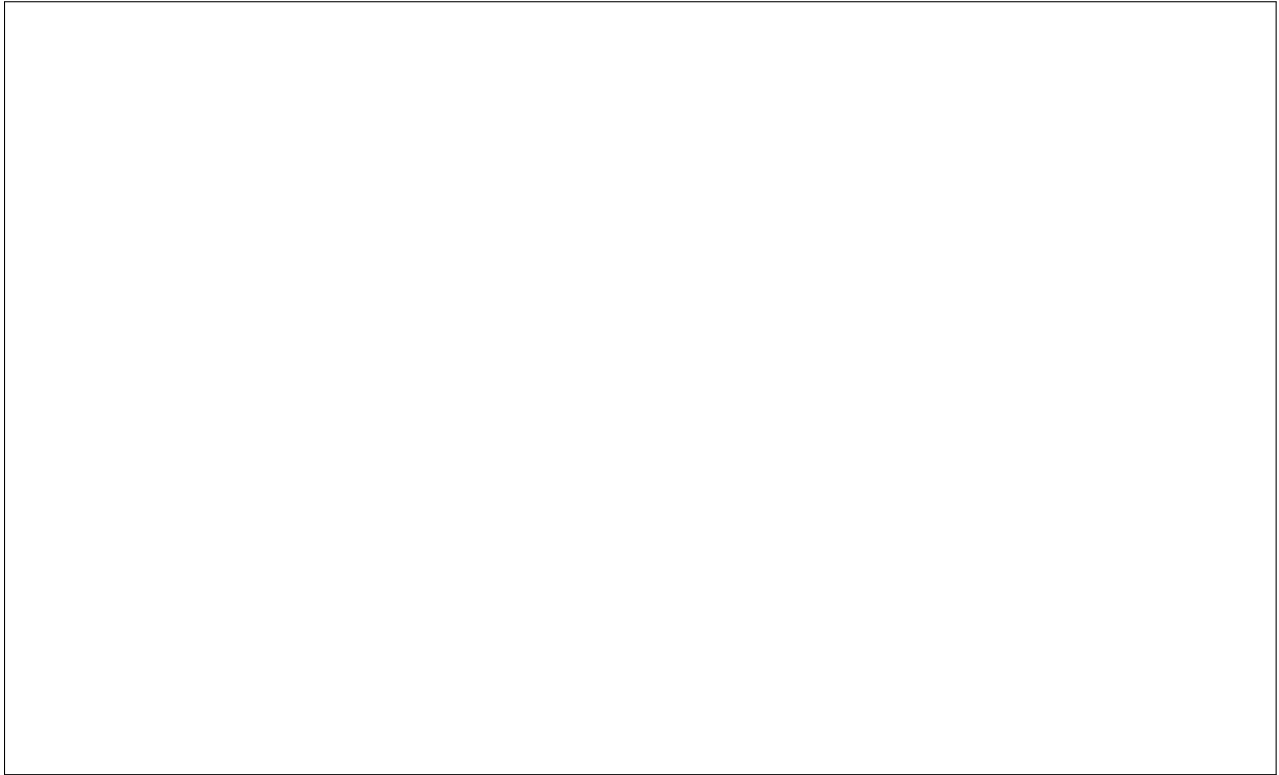
- $(a, b)$  is an **absolute minimum** if  $f(a, b) \leq f(x, y)$  for all  $(x, y)$  in the domain of  $f$
- $(a, b)$  is an **absolute maximum** if  $f(a, b) \geq f(x, y)$  for all  $(x, y)$  in the domain of  $f$
- Every absolute minimum is a local minimum
- However, a local minimum is not necessarily an absolute minimum!



- Same statements apply for absolute/local maxima

## 2 Optimization with 1 equality constraint

**Example 2.** Find the shortest distance from the point  $(1, 0, -2)$  to the plane  $x + 2y + z = 4$ .



**Example 3.** A rectangular box is to be made from  $12 \text{ m}^2$  of cardboard. Find the maximum volume of such a box.



**Example 4.** Find the maximum volume of a rectangular box inscribed in a sphere of radius 1.

