SM223 – Calculus III with Optimization Fall 2012
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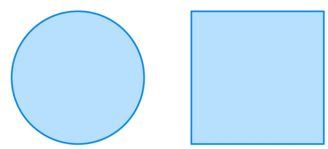
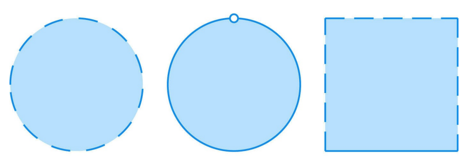
Lesson 34. Constrained Optimization, cont.

- Last time: optimization with 1 equality constraint:
  - Solve for 1 variable in the equality constraint
  - Substitute into objective function
  - Find critical points of objective function
  - Find absolute minima/maxima among critical points
- Today: optimization over a closed, bounded set:
  - A set in $\mathbb{R}^2$ is **closed** if it contains all its boundary points
  - A set in $\mathbb{R}^2$ is **bounded** if it is contained within some disk (i.e., it is finite in extent)



(a) Closed sets



(b) Sets that are not closed

- Finding the absolute minimum and maximum values of a continuous function f on a closed bounded set C:
  1. Draw a picture of C
  2. Find the values of f at the critical points of f in C
  3. Find the extreme values of f on the boundary of C
  4. Largest value from steps 2 and 3 = absolute maximum value
     Smallest value from steps 2 and 3 = absolute minimum value

Example 1. Find the absolute maximum and minimum values of $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $C = \{(x, y) \mid 0 \le x \le 3, 0 \le y \le 2\}$.

Example 2. Find the absolute maximum and minimum values of $f(x, y) = xy^2$ on $C = \{(x, y) \mid x \geq 0, y \geq 0, x^2 + y^2 \leq 4\}$.