SM223 – Calculus III with Optimization — Fall 2012
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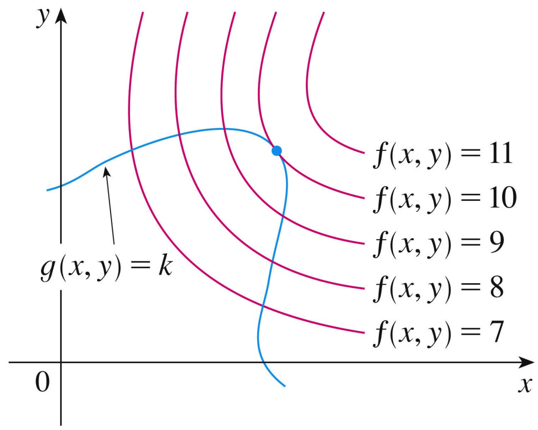
Lesson 36. Lagrange Multipliers

1 Today...

- Another way to solve optimization problems with one equality constraint

2 Lagrange multipliers for optimization with one equality constraint

- Convention: "maximum" and "minimum" refer to "absolute maximum" and "absolute minimum" resp.
- Idea:



  - Maxima and minima occur when the level curves of $f(x,y)$ and the constraint $g(x,y)$ have a common tangent line
  - In other words, the gradients of f and g are parallel:

- **Method of Lagrange multipliers for optimization with one equality constraint**
  - To find the maximum and minimum values of $f(x,y)$ subject to the constraint $g(x,y)=k$:
    1. Find all values of x, y, λ such that

       or equivalently

    2. Evaluate f at all the points (x,y) you found in step 1.
       - Largest of these values = maximum value of f
       - Smallest of these values = minimum value of f
  - (Assumes extreme values exist and $\nabla g \neq \vec{0}$ on the curve $g(x,y)=k$)
  - Works in a similar way for finding max/min values of $f(x,y,z)$ subject to $g(x,y,z)=k$
  - Suggestion: use w for λ on the calculator

Example 1. Find the absolute maximum and minimum values of $f(x, y) = y^2 - x^2$ on the ellipse $x^2 + 4y^2 = 4$.

Example 2. Find the absolute maximum and minimum values of $f(x, y) = xyz$ subject to the constraint $x + 2y + 3z = 6$.

Example 3. Find the volume of the largest rectangular box in the first orthant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.