SM223 – Calculus III with Optimization Fall 2012
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Lesson 37. Lagrange Multipliers, cont.

1 Today...

- Lagrange multipliers and optimization over closed bounded sets
- Lagrange multipliers for optimization with two equality constraints
- Homework review

2 Warm up

Example 1. Find the absolute minimum and maximum values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

3 Lagrange multipliers and optimization over closed bounded sets

- Recall: to find the absolute minimum and maximum values of a continuous function f on a closed bounded set C:
  1. Draw a picture of C
  2. Find the values of f at the critical points of f in C
  3. **Find the extreme values of f on the boundary of C**
  4. Largest value from steps 2 and 3 = absolute maximum value
     Smallest value from steps 2 and 3 = absolute minimum value
- If C can be expressed using a single inequality, Step 3 can be accomplished using Lagrange multipliers

Example 2. Find the absolute minimum and maximum values of the function $f(x, y) = x^2 + 2y^2$ on the disk $x^2 + y^2 \leq 1$.

4 Lagrange multipliers for optimization with two equality constraints

- Method of Lagrange multipliers for two constraints
  - To find the minimum and maximum values of $f(x, y, z)$ subject to the constraints $g(x, y, z) = k$ and $h(x, y, z) = \ell$:
    1. Find all values of x, y, z, λ, μ such that

       or equivalently

    2. Evaluate f at all the points (x, y, z) you found in step 1.
       - Largest of these values = maximum value of f
       - Smallest of these values = minimum value of f
  - (Assumes extreme values exist, ∇g and ∇h are not zero and not parallel on the constraints)
  - Suggestion: use w for λ and v for μ on the calculator

Example 3. Find the absolute minimum and maximum values of $f(x, y, z) = x + 2y + 4z$ subject to the constraints $x - y + z = 1$ and $x^2 + y^2 = 1$.

5 Homework review

Example 4. Find the absolute minimum and maximum values of $f(x, y) = e^{xy}$ subject to the constraint $x^3 + y^3 = 16$.