

Lesson 42. Iterated Integrals

- **Partial integration** with respect to x : $\int_a^b f(x, y) dx$
 - Regard y as a constant (i.e., fixed, coefficient, etc.)
 - Integrate $f(x, y)$ with respect to x from $x = a$ to $x = b$
 - Results in an expression in terms of y
- Partial integration with respect to y defined in a similar way
- **Iterated integrals**: work from the inside out
 - $\int_c^d \int_a^b f(x, y) dx dy = \int_c^d \left[\int_a^b f(x, y) dx \right] dy$
 - ◊ Integrate first with respect to x from $x = a$ to $x = b$ (keeping y constant)
 - ◊ Integrate resulting expression in y with respect to y from $y = c$ to $y = d$
 - $\int_a^b \int_c^d f(x, y) dy dx = \int_a^b \left[\int_c^d f(x, y) dy \right] dx$

◊ Integrate first with respect to from

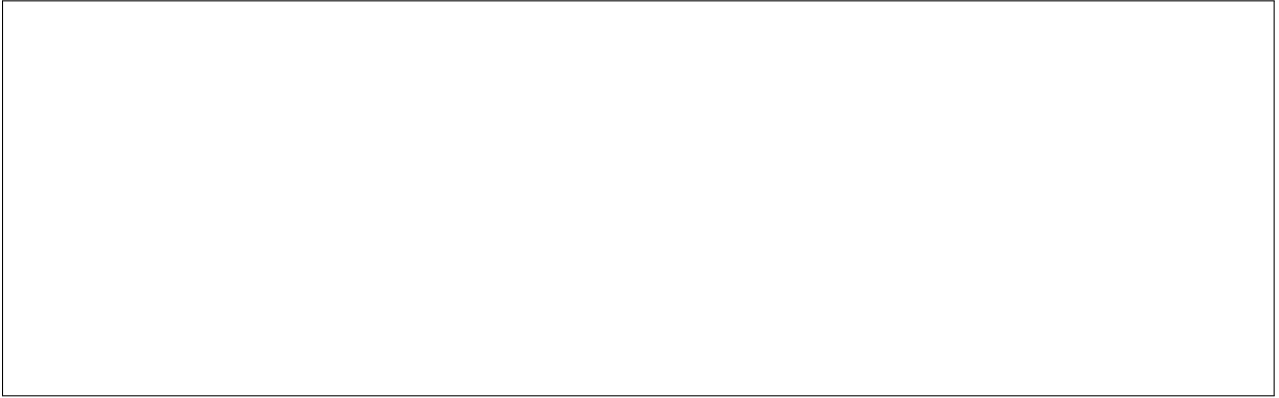
◊ Integrate resulting expression in with respect to from

Example 1. Evaluate: (a) $\int_0^3 \int_1^2 x^2 y dy dx$, (b) $\int_1^2 \int_0^3 x^2 y dx dy$

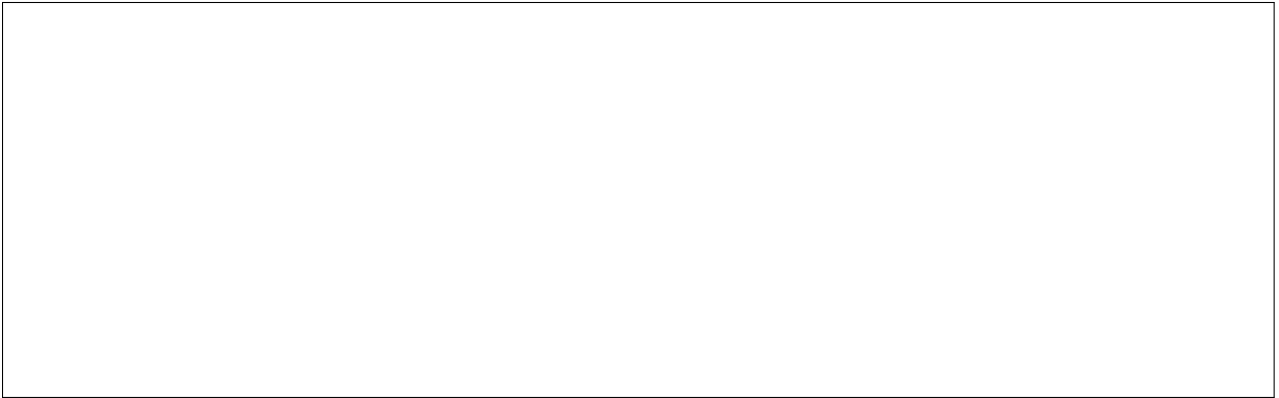
- **Fubini's theorem for rectangles.** If $R = [a, b] \times [c, d]$, then:

- (f needs to satisfy some conditions, e.g. f is continuous on R)
- Double integrals over rectangles can be evaluated using iterated integrals
- Order of integration does not matter!

Example 2. Evaluate $\iint_R (x - 3y^2) dA$, where $R = [0, 2] \times [1, 2]$.



Example 3. Find the volume of the solid that is bounded by the surface $x^2 + 2y^2 + z = 16$, the planes $x = 2$ and $y = 2$, and the three coordinate planes.



Example 4. Evaluate $\int_0^1 \int_0^1 ye^{xy} dy dx$.

