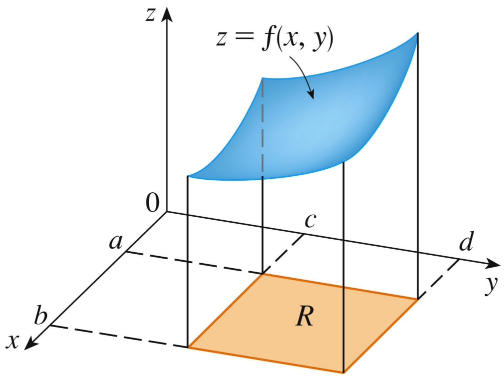
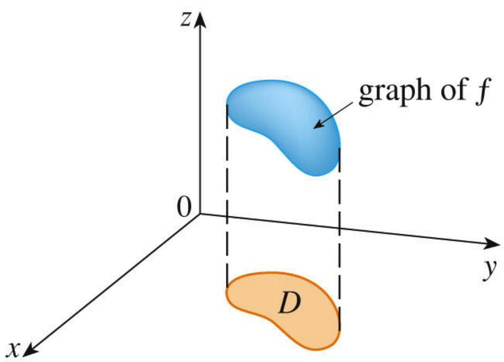
Lesson 43. Double Integrals Over General Regions

1 Last time: rectangles



- Rectangle $R = [a, b] \times [c, d]$
  $= \{(x, y) \mid a \leq x \leq b, c \leq y \leq d\}$
- $$\iint_R f(x, y)\, dA = \int_a^b \int_c^d f(x, y)\, dy\, dx = \int_c^d \int_a^b f(x, y)\, dx\, dy$$

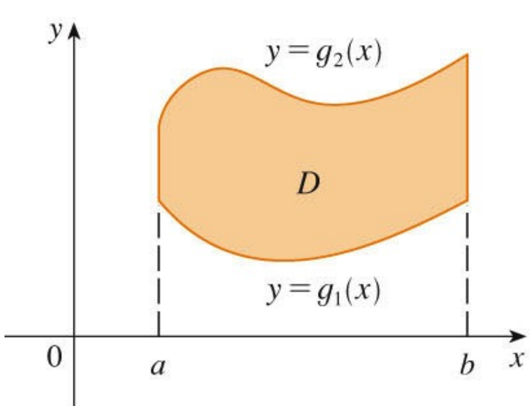
2 General regions

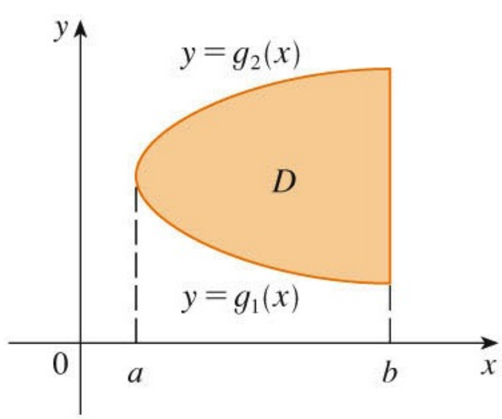


- How about general regions D?
- Intuition: if $f(x, y) \geq 0$, double integral still represents volume of solid between D and graph of f
- We focus on two types of regions

- **Type I regions**: lies between the graphs of two continuous functions of x, that is

  - Examples:



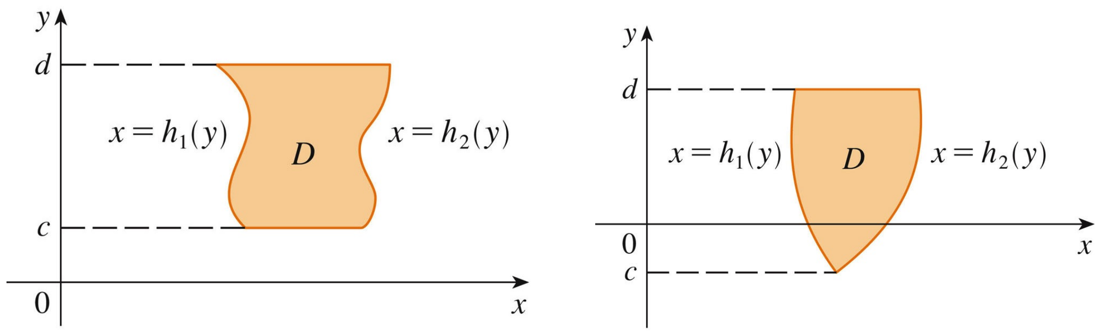


- If D is a type I region (and f is continuous on D), then

- In the inner integral, x is regarded a constant in $f(x, y)$ and the limits of integration

- **Type II regions**: lies between the graphs of two continuous functions of y, that is

  - Examples:



  - If D is a type II region (and f is continuous on D), then

  - In the inner integral, y is regarded a constant in $f(x, y)$ and the limits of integration

Example 1. Find $\iint_D (x + 2)\, dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

Example 2. Find $\iint_D (x - y)\, dy\, dx$, where $D = \{(x, y) \mid 0 \le x \le 1, 2x \le y \le 2\}$.

Example 3. Evaluate $\iint_D (2 - 2x - y)\, dA$, where D is the triangular region with vertices $(0, 0)$, $(2, 0)$, $(1, 1)$.

Example 4. Sketch the region of integration of $\int_0^2 \int_{x^2}^{2x} (x^2 + y^2)\, dy\, dx$. Change the order of integration.