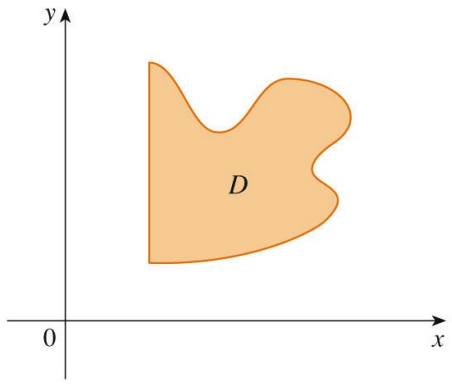
Lesson 44. Double Integrals Over General Regions, cont.

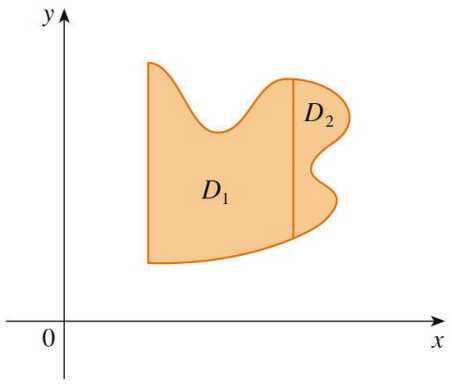
1 Properties of double integrals

- If $D = D_1 \cup D_2$, where D_1 and D_2 don't overlap except perhaps on their boundaries, then

$$\iint_D f(x,y)\,dA = \iint_{D_1} f(x,y)\,dA + \iint_{D_2} f(x,y)\,dA$$



(a) D is neither type I nor type II.



(b) $D = D_1 \cup D_2$, D_1 is type I, D_2 is type II.

Example 1. Write $\iint_D (2 - 2x - y)\,dA$ as the sum of 2 type I region iterated integrals, where D is the triangular region with vertices $(0,0)$, $(2,0)$, and $(1,1)$.

2 Problems

Problem 1. Evaluate $\iint_D x \cos y \, dA$, where D is bounded by $y = 0$, $y = x^2$, and $y = 1$.

Problem 2. Let D be the region enclosed by the lines $y = x$, $y = 0$, and $x = 1$. Set up $\iint_D x \, dA$ using both orders of integration. Evaluate the double integral using the order of integration of your choice.

Problem 3. Set up a double integral that represents the volume of the solid enclosed by $z = 1 - x^2 - y^2$ and $z = 0$ (i.e., the xy-plane). Use a calculator to evaluate the integral. Hints:

- Write an equation for where these two surfaces intersect. Graph this equation in the xy-plane. This should give you an idea of how to set up your limits of integration.
- TI-nspire CX tips:
  - Press the  button to get a template for a definite integral.
  - You can put an integral template within another integral template to do double integrals.

Problem 4. Thought exercise: Let D be some region in the xy-plane. What does $\iint_D 1 \, dA$ represent? Explain.