SM223 – Calculus III with Optimization
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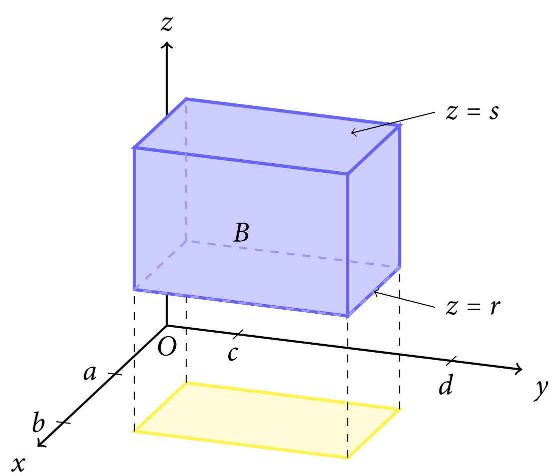
Lesson 48. Triple Integrals

- Functions of one variable: intervals of integration
- Functions of two variables: 2D regions of integration
- Functions of three variables: 3D regions of integration

1 Triple integrals over rectangular boxes

- **Fubini's theorem for triple integrals.** Let $B = \{(x, y, z) \mid a \le x \le b,\ c \le x \le d,\ r \le z \le s\}$. Then

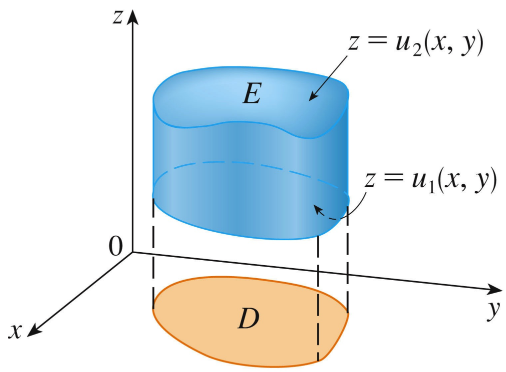
  - (f continuous on B)
  - Integrate **from the inside out**
  - When all limits of integration are constant, we can integrate in any order



Example 1. Evaluate the triple integral $\iiint_B x \, dV$, where B is the rectangular box given by $B = \{(x, y, z) \mid 0 \le x \le 1, -1 \le y \le 2, 0 \le z \le 3\}$.

2 Triple integrals over general bounded 3D regions: Type A

- **Type A 3D region:** between two continuous functions of x and y



  - E is the 3D region
  - D is the projection (shadow) of E onto the xy-plane
  - If E is a type A region, then

  - (f, u_1, u_2 continuous)
  - Integration from the inside out
  - Double integral over D can be done using previous techniques (e.g. Type I or II region)

Example 2. Express $\iiint_E xz\,dV$ as an iterated integral, where E lies below the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = x^2$ and $y = x$.

Example 3. Express $\iiint_E \sin(x + yz)\,dV$ as an iterated integral, where E lies below the surface $z = 1 + x^2 + 4y^2$ and above the region in the xy-plane bounded by the curves $x = 2y$, $x = 0$, and $y = 1$.

Example 4. Express $\iiint_E e^z \, dV$ as an iterated integral, where E is the solid tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$.