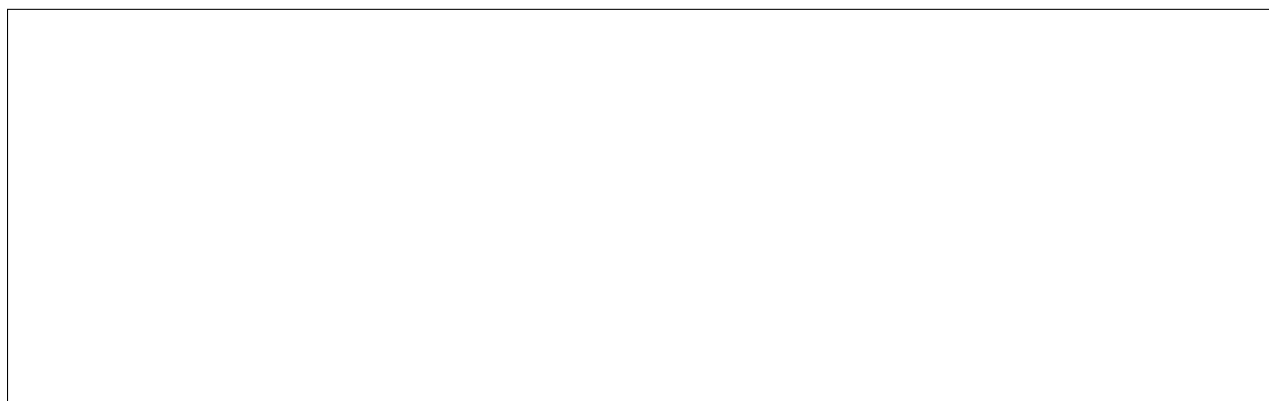


## Lesson 49. Triple Integrals, cont.

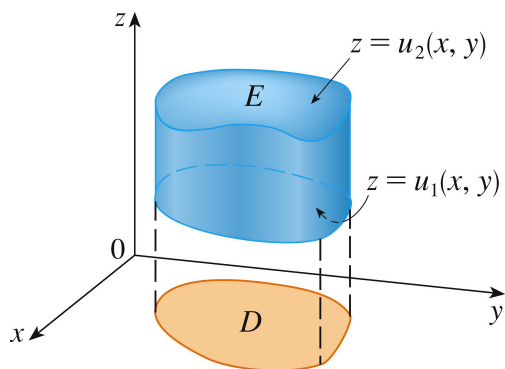
### 0 Warm up

**Example 1.** Express  $\iiint_E y\sqrt{z} \, dV$  as an iterated integral, where  $E$  is the tetrahedron enclosed by the coordinate planes and the plane  $2x + y + z = 4$ .



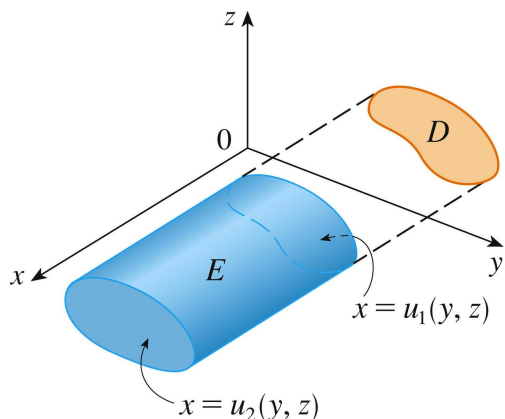
### 1 Types of 3D regions of integration

- **Type A 3D region:** between two continuous functions of  $x$  and  $y$



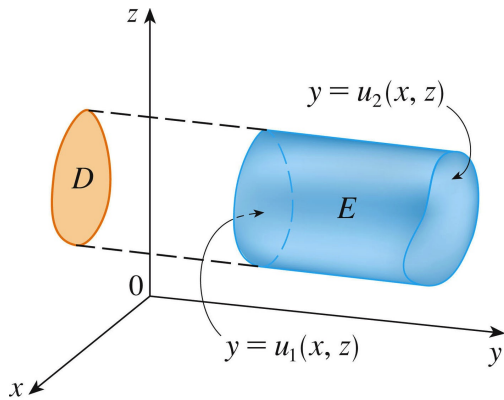
$$\begin{aligned} \iiint_E f(x, y, z) \, dV \\ = \iint_D \left[ \int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z) \, dz \right] dA \end{aligned}$$

- **Type B 3D region:** between two continuous functions of  $y$  and  $z$



$$\begin{aligned} \iiint_E f(x, y, z) \, dV \\ = \iint_D \left[ \int_{u_1(y,z)}^{u_2(y,z)} f(x, y, z) \, dx \right] dA \end{aligned}$$

- **Type C 3D region:** between two continuous functions of  $x$  and  $z$



$$\begin{aligned} \iiint_E f(x, y, z) dV \\ = \iint_D \left[ \int_{u_1(x, z)}^{u_2(x, z)} f(x, y, z) dy \right] dA \end{aligned}$$

## 2 Finding volume

- $\int_a^b 1 dx =$

- $\iint_D 1 dA =$

- $\iiint_E 1 dV =$

## 3 Examples and Problems

**Example 2.** Express  $\iiint_E y\sqrt{z} dV$  as an iterated integral, where  $E$  is the tetrahedron enclosed by the coordinate planes and the plane  $2x + y + z = 4$ . Consider  $E$  as a type B and a type C region.

**Example 3.** Evaluate  $\iiint_E \sqrt{x^2 + z^2} dV$  where  $E$  is the region bounded by the paraboloid  $y = x^2 + z^2$  and the plane  $y = 4$ .

