

Lesson 51. Mass and Center of Mass, Revisited

1 Definitions

- Suppose a solid object occupies a region E in space
- Let $\rho(x, y, z)$ = density of the object at (x, y, z)
- The **mass** of the object is

- The **center of mass** of the object is located at $(\overline{x}, \overline{y}, \overline{z})$, where

2 Problems

Example 1. Set up integrals to find the mass and center of mass of the solid E, where E is the solid above the xy-plane and bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = y$ and $z = 0$, and the density function is $\rho(x, y, z) = 1 + x + y + z$.

Example 2. Let E be the solid bounded by the parabolic cylinder $x = y^2$ and the planes $z = x$, $z = 0$, and $x = 1$, and let the density function be $\rho(x, y, z) = y^2$.

a. Set up integrals for the mass and center of mass of E.
b. Use your calculator to evaluate the integrals you set up in part a.

Example 3. Let E be the solid bounded by the sphere $x^2 + y^2 + z^2 = 1$ and the xy-plane, and let the density function be $\rho(x, y, z) = (x - 1)^2$.

a. Set up integrals for the mass and center of mass of E.
b. Use your calculator to evaluate the integrals you set up in part a.