List of formulas for the Final Exam, SM223, Fall 2020

- Equation of a sphere with center $C(h, k, l)$ and radius r
$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2.$$
- If θ is the angle between the vectors **a** and **b**, then
$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$
- The **direction cosines** of the the vector **a**:
$$\cos\alpha = \frac{a_1}{|\mathbf{a}|},\ \cos\beta = \frac{a_2}{|\mathbf{a}|},\ \cos\gamma = \frac{a_3}{|\mathbf{a}|}.$$
- Scalar projection of **b** onto **a**:
$$\text{comp}_a \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$$
- Vector projection of **b** onto **a**
$$\text{proj}_a b = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a}$$
- The work done by a constant force **F** along the displacement vector **D** is
$$\mathbf{F} \cdot \mathbf{D}.$$
- $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **cross product** of **a** and **b** is the vector
$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
- If v is a vector parallel to L and let r_0 is the position vector of P_0, the **vector equation** of the line L through P is
$$\mathbf{r} = \mathbf{r_0} + t\mathbf{v},$$
where t is the **parameter.**

- The **parametric equations** of a line**:**

$$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}$$

- A plane is determined by a point $P_0(x_0, y_0, z_0)$ in the plane and a vector $\mathbf{n}$ that is orthogonal to the plane.

$$\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$$

- The **scalar equation** of the plane through $P_0(x_0, y_0, z_0)$ with normal vector $\boldsymbol{n} = \langle a, b, c \rangle$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$

- The **linear equation** of a plane

$$ax + by + cz + d = 0$$

- The **unit tangent vector** for a vector function $\mathbf{r}(t)$ at t is

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

- The **length** of a space curve $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ is

$$L = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2} \mathrm{d}t$$

- Let $\mathbf{r}(t)$ be a space curve. The **velocity vector** $\mathbf{v}(t)$ at time t is

$$\mathbf{v}(t) = \lim_{h \to 0} \frac{\mathbf{r}(t + h) - \mathbf{r}(t)}{h} = \mathbf{r}^{'}(t).$$

- The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is constant.

- An equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$ is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

- The linear function whose graph is this tangent plane

$$L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

  is called the **linearization** of f at (a, b)

- The approximation

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

  is called the **linear approximation** or the **tangent plane approximation** of f at (a,b).

- **Chain Rule, case 1:** Suppose that $z = f(x,y)$ is a differentiable function of x and y, where $x = g(t)$ and $y = h(t)$ are both differentiable functions of t. Then z is a differentiable function of t and

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial f}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

- **Chain Rule, case 2:** Suppose that $z = f(x,y)$ is a differentiable function of x and y, where $x = g(s,t)$ and $y = h(s,t)$ are differentiable functions of s and t. Then

$$\begin{aligned} \frac{\partial z}{\partial s} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \\ \frac{\partial z}{\partial t} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} \end{aligned}$$

- If f is a function of two variables x and y, then the **gradient** of f is the vector function ∇f defined by

$$\nabla f(x,y) = \langle f_x(x,y), f_y(x,y)\rangle = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}.$$

- **The tangent plane to the level surface** $F(x,y,z) = k$ **at** $P(x_0, y_0, z_0)$ is the plane that passes through P and has normal vector $\nabla F(x_0, y_0, z_0)$.

- The **normal line** to S at P is the line passing through P and perpendicular to the tangent plane.

- **Second Derivative Test:** Suppose the second partial derivatives of f are continuous on a disk with center (a,b), and suppose that $f_x(a,b) = 0$ and $f_y(a,b) = 0$. Let

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - (f_{xy})^2.$$

  1. If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum.
  2. If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum.
  3. If $D < 0$, then $f(a,b)$ is not a local maximum or minimum. In this case the point (a,b) is called a **saddle point** of f.

- **Method of Lagrange Multipliers:** To find the maximum and minimum values of $f(x,y,z)$ subject to the constraint $g(x,y,z) = k$:

1. Find all values of x, y, z, and λ such that

$$\nabla f(x,y,z) = \lambda \nabla g(x,y,z) \text{ and } g(x,y,z) = k.$$

2. Evaluate f at all the points (x, y, z) that result from step 1.
   - The largest of these values is the maximum value of f.
   - The smallest is the minimum value of f.

- **Midpoint Rule:**

$$\iint_R f(x,y)dA \simeq \sum_{i=1}^{m}\sum_{j=1}^{n} f(\overline{x}_i, \overline{y}_j)\Delta A,$$

where $\overline{x}_i$ is the midpoint of $[x_{i-1}, x_i]$ and $\overline{y}_j$ is the midpoint of $[y_{j-1}, y_j]$.

- **Integral using polar coordinates:** If the polar rectangle R is given by $0 \leq a \leq r \leq b$, $\alpha \leq \theta \leq \beta$, where $0 \leq \beta - \alpha \leq 2\pi$, then

$$\iint_R f(x,y)dA = \int_{\alpha}^{\beta}\int_{a}^{b} f(r\cos\theta, r\sin\theta)\,r\,drd\theta.$$

- The total mass m of the lamina equals

$$m = \iint_D \rho(x,y)dA.$$

- The **moment** of the entire lamina **about the x-axis** is:

$$M_x = \iint_D y\rho(x,y)dA.$$

- The **moment** of the entire lamina **about the y-axis** is:

$$M_y = \iint_D x\rho(x,y)dA.$$

- The coordinates $(\overline{x}, \overline{y})$ of the center of the mass of a lamina are)

$$\overline{x} = \frac{M_y}{m} = \frac{\iint_D x\rho(x,y)dA}{\iint_D \rho(x,y)dA},$$
$$\overline{y} = \frac{M_x}{m} = \frac{\iint_D y\rho(x,y)dA}{\iint_D \rho(x,y)dA}.$$