Lesson 18. Tangent Planes and Normal Lines

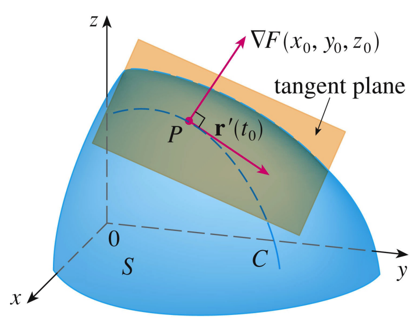
0 Warm up

Example 1. Let P be the point $(2, 0, 1)$ and $\vec{v} = \langle 1, -2, 5 \rangle$.

a. Find parametric equations of the line that passes through P and is parallel to $\vec{v}$.
b. Find an equation of the plane through point P with normal vector $\vec{v}$.

1 Tangent planes and normal lines in 3D

- Consider a surface with equation $F(x, y, z) = k$
- The gradient $\nabla F(x_0, y_0, z_0)$ is [ ] to the surface at (x_0, y_0, z_0)



- The **tangent plane to the surface** $F(x, y, z) = k$ at (x_0, y_0, z_0) is the plane that
  - passes through (x_0, y_0, z_0) and
  - has normal vector $\nabla F(x_0, y_0, z_0)$
- Equation of tangent plane to $F(x, y, z) = k$ at (x_0, y_0, z_0):

Example 2. Find an equation of the tangent plane to the ellipsoid $\frac{x^2}{9} + y^2 + \frac{z^2}{4} = 3$ at the point $(-3, 1, -2)$.

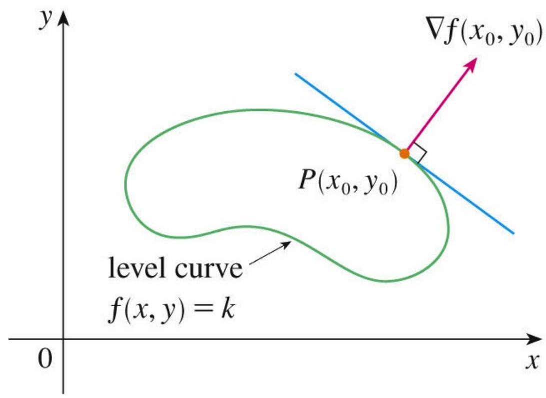
Example 3. Find an equation of the tangent plane to the surface $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.

- The **normal line to the surface** $F(x, y, z) = k$ at point (x_0, y_0, z_0) is the line that
  - passes through (x_0, y_0, z_0) and
  - is perpendicular to the tangent plane (i.e., is parallel to $\nabla F(x_0, y_0, z_0)$)
- Parametric equations of the normal line to $F(x, y, z) = k$ at (x_0, y_0, z_0):

Example 4. Find the normal line to the ellipsoid $\frac{x^2}{9} + y^2 + \frac{z^2}{4} = 3$ at the point $(-3, 1, -2)$.

2 Tangent lines in 2D

- The **tangent line to the curve** $f(x, y) = k$ at (x_0, y_0) is given by



Example 5. Let $g(x, y) = x^2 + y^2 - 4x$. Find the tangent line to the curve $g(x, y) = 1$ at the point $(1, 2)$.