5 Generalizations

- All of the above generalizes naturally to $\mathbb{R}^3$:

$$|\langle a_1, a_2, a_3\rangle| = \sqrt{a_1^2 + a_2^2 + a_3^2} \qquad \langle a_1, a_2, a_3\rangle + \langle b_1, b_2, b_3\rangle = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3\rangle$$

$$c\langle a_1, a_2, a_3\rangle = \langle ca_1, ca_2, ca_3\rangle \qquad \langle a_1, a_2, a_3\rangle - \langle b_1, b_2, b_3\rangle = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3\rangle$$

- Algebraically, vectors behave a lot like scalars, e.g.

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \qquad c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b} \qquad (c + d)\vec{a} = c\vec{a} + d\vec{a}$$

- See p. 802 of Stewart for a fuller list

6 Standard basis vectors and unit vectors

- **Standard basis vectors** in $\mathbb{R}^3$:

$\vec{i} = \boxed{\langle 1, 0, 0\rangle}$ $\quad \vec{j} = \boxed{\langle 0, 1, 0\rangle}$ $\quad \vec{k} = \boxed{\langle 0, 0, 1\rangle}$

- We can write any vector as the sum of scalar multiples of standard basis vectors:

$$\boxed{\langle a_1, a_2, a_3\rangle = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}}$$

- A **unit vector** is a vector with length 1
  - For example, $\vec{i}, \vec{j}, \vec{k}$ are all unit vectors

- The unit vector that has the same direction as $\vec{a}$ (assuming $\vec{a} \neq \vec{0}$) is $\boxed{\dfrac{\vec{a}}{|\vec{a}|}}$

Example 5. Let $\vec{a} = 4\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{b} = \vec{i} + 2\vec{k}$.

a. Write $\vec{a} - 2\vec{b}$ in terms of $\vec{i}, \vec{j}, \vec{k}$.
b. Find a unit vector in the direction of $\vec{a} - 2\vec{b}$.

a. $\vec{a} - 2\vec{b} = 4\vec{i} - \vec{j} + 2\vec{k} - 2(\vec{i} + 2\vec{k})$
$= 4\vec{i} - \vec{j} + 2\vec{k} - 2\vec{i} - 4\vec{k}$
$= 2\vec{i} - \vec{j} - 2\vec{k} = \langle 2, -1, -2\rangle$

b. $|\vec{a} - 2\vec{b}| = |\langle 2, -1, -2\rangle|$
$= \sqrt{2^2 + (-1)^2 + (-2)^2}$
$= \sqrt{9} = 3$

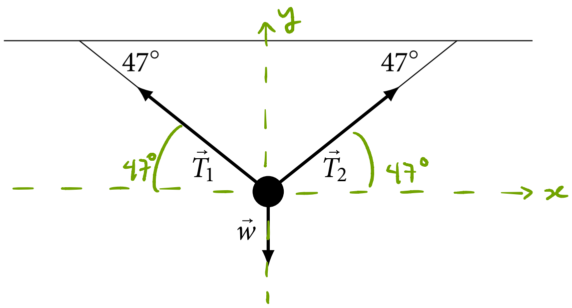
$\Rightarrow$ unit vector in the same direction as $\vec{a} - 2\vec{b} = \dfrac{\langle 2, -1, -2\rangle}{3}$
$= \langle \frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\rangle$

- Note: all of this applies to vectors in $\mathbb{R}^2$ in a similar way

7 Problems with forces

- Some physics:
  - **Force** has magnitude and direction, and so it can be represented by a vector
  - Force is measured in pounds (lbs) or newtons (N)
  - If several forces are acting on an object, the **resultant force** experienced by the object is the sum of these forces

Example 6. A weight $\vec{w}$ counterbalances the tensions (forces) in two wires as shown below:



The tensions $\vec{T}_1$ and $\vec{T}_2$ both have a magnitude of 20lb. Find the magnitude of the weight $\vec{w}$.

$\vec{T}_1 = \langle -20\cos 47, 20\sin 47 \rangle$ $\qquad$ $\vec{T}_2 = \langle 20\cos 47, 20\sin 47 \rangle$

Resultant force $= \vec{0}$

$\vec{0} = \vec{T}_1 + \vec{T}_2 + \vec{w} \implies \vec{w} = -\vec{T}_1 - \vec{T}_2$

$= \langle 0, -40\sin 47 \rangle$

$\implies |\vec{w}| = \sqrt{0^2 + (-40\sin 47)^2} = \sqrt{(40\sin 47)^2} = 40\sin 47$

- Note: if an object has a mass of m kg, then it has a weight of mg N, where $g = 9.8$