6 Practice!

Example 8. Find the scalar projection and vector projection of $\vec{b} = \langle 1, 1, 2\rangle$ onto $\vec{a} = \langle -2, 3, 1\rangle$.

$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{-2+3+2}{\sqrt{4+9+1}} = \frac{3}{\sqrt{14}}$$

$$\text{proj}_{\vec{a}}\vec{b} = \left(\text{comp}_{\vec{a}}\vec{b}\right)\frac{\vec{a}}{|\vec{a}|} = \frac{3}{\sqrt{14}}\,\frac{\langle -2,3,1\rangle}{\sqrt{14}} = \frac{3}{14}\langle -2,3,1\rangle = \left\langle -\frac{6}{14}, \frac{9}{14}, \frac{3}{14}\right\rangle$$

Example 9. Find a unit vector that is orthogonal to both $\overbrace{\langle 2, 0, -1\rangle}^{\vec{a}}$ and $\overbrace{\langle 0, 1, -1\rangle}^{\vec{b}}$.

Let $\vec{u} = \langle u_1, u_2, u_3\rangle$ be such a unit vector.

Then $\vec{u}$ must satisfy:

$$\begin{matrix}\vec{a}\cdot\vec{u} = 0\\ \vec{b}\cdot\vec{u} = 0\end{matrix} \Rightarrow \begin{cases} 2u_1 - u_3 = 0 \\ u_2 - u_3 = 0\end{cases}$$

$$\Rightarrow \begin{cases} u_1 = \dfrac{u_3}{2} \\ u_2 = u_3\end{cases}$$

Also, since $\vec{u}$ is a unit vector:

$$u_1^2 + u_2^2 + u_3^2 = 1$$

$$\Rightarrow \frac{1}{4}u_3^2 + u_3^2 + u_3^2 = 1$$

$$\Rightarrow \frac{9}{4}u_3^2 = 1 \quad \Rightarrow u_3^2 = \frac{4}{9}$$

$$\Rightarrow u_3 = \frac{2}{3} \quad \text{(ignore negative part)}$$

$$\Rightarrow \vec{u} = \left\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right\rangle$$

Example 10. Determine whether the given vectors are orthogonal, parallel, or neither:

a. $\vec{a} = \langle 4, 5, -2\rangle$, $\vec{b} = \langle 3, -1, 5\rangle$
b. $\vec{u} = 9\vec{i} - 6\vec{j} + 3\vec{k}$, $\vec{v} = -6\vec{i} + 4\vec{j} - 2\vec{k}$

a. $\vec{a}$ and $\vec{b}$ are not parallel, since they are not scalar multiples of each other

$\vec{a}\cdot\vec{b} = 12 - 5 - 10 = -3$

$\Rightarrow \vec{a}$ and $\vec{b}$ are not orthogonal

b. $\vec{u}$ and $\vec{v}$ are parallel, since $\vec{u} = -\frac{3}{2}\vec{v}$

$\vec{u}\cdot\vec{v}$ therefore are not orthogonal