Name: Notes

SM275 · Mathematical Methods for Economics Fall 2019 · Uhan

# Exam 2 – 28 October 2019

## Instructions

- You have until the end of the class period to complete this exam.
- You may not use your calculator.
- You may not consult any other outside materials (e.g. notes, textbooks, homework, computer).
- **Show all your work.** To receive full credit, your solution must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.

| Problem | Weight | Score |
|---|---|---|
| 1 | 1 | |
| 2 | 1 | |
| 3 | 1 | |
| 4 | 1 | |
| 5 | 1 | |
| 6 | 1 | |
| 7 | 1 | |
| 8 | 2 | |
| 9 | 1 | |
| 10 | 2 | |
| 11 | 1 | |
| 12 | 1 | |
| 13 | 1 | |
| 14 | 1 | |
| 15 | 2 | |
| 16 | 2 | |
| Total | | / 200 |

For the problems on this page, let

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -2 \\ 0 & 3 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 4 \\ 3 & 5 \\ -2 & 0 \end{bmatrix}$$

If the quantity you are asked to compute is undefined, briefly explain why.

**Problem 1.** Compute $A - 2C$.

**Problem 2.** Compute $AB$.

**Problem 3.** Compute $AC$.

**Problem 4.** Compute $B^{-1}$.

- You can compute this using the formula for an inverse of a $2 \times 2$ matrix (page 2 of Lesson 12), or using elementary row operations (page 8 of Lesson 12).

For the problems on this page, let

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

If the quantity you are asked to compute is undefined, briefly explain why.

**Problem 5.** Compute $A^T$.

**Problem 6.** Compute $AB$.

- Note that $B$ is an identity matrix. See Section 10 of Lesson 11.

**Problem 7.** Compute $BA^TC$. What size is $BA^TC$?

- Note that $C$ is a null matrix. See Section 11 of Lesson 11.

Consider the system of linear equations below.

$$\begin{aligned} 2x_1 + \ \ 4x_2 - 2x_3 + 2x_4 + 4x_5 &= 2 \\ x_1 + \ \ 2x_2 - \ \ x_3 + 2x_4 \qquad\ \ &= 4 \\ 3x_1 + \ \ 6x_2 - 2x_3 + \ \ x_4 + 9x_5 &= 1 \\ 5x_1 + 10x_2 - 4x_3 + 5x_4 + 9x_5 &= 9 \end{aligned}$$

The reduced row echelon form of the augmented matrix for this system is

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**Problem 8.** What are the solutions of this system? Write the solutions in vector form. If there are no solutions, simply state so.

- Take a look at Example 9 in Lesson 12, as well as Problems 3.2b and 3.2e assigned for homework, for similar problems.

**Problem 9.** How many solutions does this system have?

- I did not grade your explanations for this problem.
- Some of you wrote that the system has an infinite number of solutions because the RREF contains the equation $0 = 0$. This isn't completely correct.
- If the RREF has a row of the form $\begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$, then the system has no solutions because the equation corresponding to this row is $0 = 1$.
- If there is no such row in the RREF:
    - If there is a free variable, then there are infinitely many solutions.
    - If there are no free variables, then there is exactly one solution.

For this page, let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 4 & 0 \end{bmatrix}$.

**Problem 10.** Compute $A^{-1}$. (You may assume it exists.)

- Most of you started correctly.
- Be careful with your arithmetic!
- Take a look at page 8 of Lesson 12 if you need help getting started.

**Problem 11.** Does $|A| = 0$? Briefly explain without computing $|A|$.

- Some of you wrote: Since there isn't one row that is a multiple of another row, therefore $|A| \neq 0$. This is a misuse of Property V from Lesson 13, which says: If one row is a multiple of another row, then $|A| = 0$.
- Take a look Section 4 of Lesson 13.

For this page, let

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 4 & 1 & 3 \\ 3 & 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -7 & 2 \\ 1 & ?? & 2 \\ 1 & 4 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 4 & ?? & 5 \\ 0 & -1 & ?? \\ 0 & 0 & 2 \end{bmatrix}$$

Note that some of the entries in the above matrices are deliberately missing.

**Problem 12.** Compute $|A|$.

**Problem 13.** Compute $|B|$.

- Take a look at Section 5 of Lesson 13.

**Problem 14.** Compute $|C|$.

- Take a look at Section 5 of Lesson 13.

**Problem 15.** Recall the national income model

$$Y = C + I_0 + G_0 \tag{1}$$
$$C = a + bY \qquad (0 < b < 1) \tag{2}$$

where

$$\begin{aligned} Y &= \text{national income} \\ C &= \text{consumer expenditure} \\ I_0 &= \text{business expenditure (i.e., investment)} \\ G_0 &= \text{government expenditure} \end{aligned}$$

Suppose $I_0 = 7$, $G_0 = 2$, $a = 3$, $b = \frac{1}{3}$. Use Cramer's rule to find the national income $Y$ and consumer expenditure $C$.

- Take a look at Section 3 and Problem 2 in Lesson 15.

**Problem 16.** Consider an economy with two industries. Industry 1 manufactures product 1, and industry 2 manufactures product 2.

Industry 1 uses 0.4 dollars of product 1 and 0.5 dollars of product 2 for every dollar of product 1 it manufactures. Industry 2 uses 0.1 dollars of product 1 and 0.3 dollars of product 2 for every dollar of product 2 it manufactures.

Consumers demand \$20,000 of product 1 and \$10,000 of product 2.

Let

$$x_1 = \text{output of industry 1, in dollars}$$
$$x_2 = \text{output of industry 2, in dollars}$$

Write the Leontief input-output matrix equation for this model — i.e., the matrix equation that ensures that each industry's output is equal to the input demand and the final demand for its product. Your answer should look like this:

$$\begin{bmatrix} \text{some matrix} \\ \text{with numbers} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \text{some other matrix} \\ \text{with numbers} \end{bmatrix}$$

- Many of you identified the input matrix $A$ correctly, but did not write the correct equation for the Leontief input-output model.
- Take a look at page 3 of Lesson 14.