Example 5. Find the particular solution to the DS $A_{n+2} = 2A_{n+1} - A_n + 3$ that satisfies $A_0 = 0$, $A_1 = -1$. What is A_{10}?

Here, $a=2$, $b=-1$, $c=3$ $\Rightarrow$ CE: $x^2 = 2x - 1$

$\Rightarrow r = s = 1$

GS: $A_n = c_1 + c_2 n + \left(\frac{3}{2}\right) n^2$

IC: $A_0 = 0 \Rightarrow 0 = c_1$

$A_1 = -1 \Rightarrow -1 = c_1 - c_2 + \frac{3}{2} \Rightarrow c_2 = \frac{5}{2}$

PS: $A_n = \frac{5}{2} n + \left(\frac{3}{2}\right) n^2$

$A_{10} = \frac{5}{2}(10) + \left(\frac{3}{2}\right) 10^2$

$= 175$

Example 6. Find the particular solution to the DS $A_{n+2} = 2A_{n+1} - A_n + 4$ that satisfies $A_0 = 3$, $A_1 = 6$. What is A_{10}?

Here, $a=2$, $b=-1$, $c=4$ $\Rightarrow$ CE: $x^2 = 2x - 1$

$\Rightarrow r = s = 1$

GS: $A_n = c_1 + c_2 n + 2n^2$

IC: $A_0 = 3 \Rightarrow 3 = c_1$

$A_1 = 6 \Rightarrow 6 = c_1 + c_2 + 2 \Rightarrow c_1 + c_2 = 4 \Rightarrow c_2 = 1$

PS: $A_n = 3 + n + 2n^2$

$A_{10} = 3 + 10 + 2(10^2)$

$= 213$