

## Lesson 9. Stability of Economic Systems

### 1 Discrete market models

- Recall the discrete market model from Lesson 5

- Variables:

$D_t$  = demand at time  $t$

$S_t$  = supply at time  $t$

$P_t$  = price at time  $t$

- Equations:

$$D_t = S_t$$

$$D_t = a - bP_t$$

$$S_t = -c + dP_{t-1}$$

where  $a, b, c, d$  are positive constants

- Dynamical system:

$$P_{t+1} = \left(-\frac{d}{b}\right)P_t + \frac{a+c}{b}$$

- General solution to DS:

$$P_t = k\left(-\frac{d}{b}\right)^t + \bar{P} \quad \text{where} \quad \bar{P} = \frac{a+c}{b+d}$$

**Example 1.** In words, what does it mean to increase  $b$ ? What does it mean to decrease  $d$ ?

- If  $|\frac{d}{b}| < 1$ , then

- If  $|\frac{d}{b}| > 1$ , then

- If  $|\frac{d}{b}| = 1$ , then

**Example 2.** Suppose the market is unstable and we would like to convert it to a stable market. What should we do?



**Example 3.** Suppose we can control the market to the extent that we can set the parameters  $a, b, c, d$  to be whatever we like. Find values of the parameters so that the equilibrium prices  $P_t$  converge to 3 as  $t \rightarrow \infty$ .



## 2 National income models

- Recall the national income model from Lesson 7

- Variables:

$T_n$  = total national income at time  $n$

$C_n$  = consumer expenditures at time  $n$

$I_n$  = private investment at time  $n$

$G_n$  = government expenditures at time  $n$

- Equations:

$$T_n = C_n + I_n + G_n$$

$$C_{n+1} = mT_n$$

$$I_{n+1} = \ell(C_{n+1} - C_n)$$

$$G_n = 1$$

where  $m, \ell$  are positive constants

- Dynamical system:

$$T_{n+2} = m(1 + \ell)T_{n+1} - m\ell T_n + 1$$

- For this second-order linear DS:

- “ $a + b$ ” =

- $r$  and  $s$  are the roots of the characteristic equation

- So when  :

- The general solution of this DS is:

- The fixed point of this DS is:

**Example 4.** Suppose we want the national income to converge to 5 as  $n \rightarrow \infty$ . Find values of  $m$  and  $\ell$  to make this happen.