

Lesson 14. Economic Applications of Linear Systems

1 The Leontief Input-Output Model

- Economy with a number of industries
- Each industry produces a single homogeneous product
- **Input demand** for each product:
 - Outputs for one industry are used as input for another industry
- **Final demand** for each product:
 - e.g. consumer households, government sector, foreign countries
- What output should each industry produce to satisfy the total demand for all products?

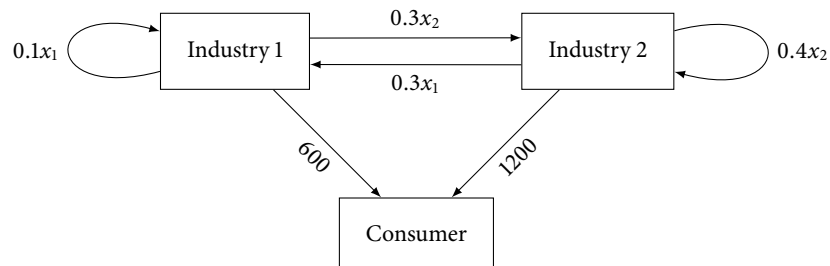
2 An example

- Economy with 2 industries/products
- Let

x_1 = output of industry 1, in dollars

x_2 = output of industry 2, in dollars

- Required inputs and demand, in dollars:



- Output of each industry must be just sufficient to meet the required inputs and demand:

- We can rewrite this system of equations in matrix form:

- Using the inverse of the coefficient matrix, we can find the required output levels x_1 and x_2 :

- Note that \$1 of product 1 requires \$0.1 of product 1 and \$0.3 of product 2

⇒ \$1 of product 1 requires $\$(1 - 0.1 - 0.3) = \0.6 of **primary inputs**

- Inputs besides those supplied by the industries in the economy, e.g. labor

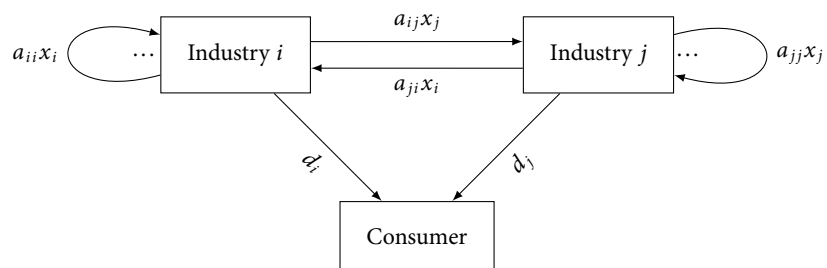
- In addition, note that \$1 of product 2 requires \$0.3 of product 1 and \$0.4 of product 2

- Therefore, \$1 of product 2 requires of primary inputs

- What is the total required amount of primary inputs for this economy?

3 Generalization to economies with n industries

- Let x_i = output of industry i , in dollars ($i = 1, \dots, n$)
- Required inputs and demand, in dollars:



- a_{ij} = dollars of product i required to produce one dollar of product j ($i = 1, \dots, n; j = 1, \dots, n$)
- d_i = final demand of product i in dollars ($i = 1, \dots, n$)

- Output of each industry must be just sufficient to meet the required inputs and demand:

- We can rewrite this system of equations in matrix form:

- Let A be the matrix of a_{ij} values – the **input matrix**
- Let D be the column vector of d_i values
- We can represent this system as

- $I - A$ is the **Leontief matrix**
- If $I - A$ is nonsingular, then we can solve for the required outputs:

- \$1 of product j requires a_{1j} of product 1, a_{2j} of product 2, ..., a_{nj} of product n
- Let a_{0j} represent the primary inputs required for \$1 of product j :

- The total required amount of primary inputs for this economy is:

Example 1. Consider an economy with 3 industries/products. Suppose the input matrix is

$$A = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.4 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.2 \end{bmatrix}$$

and the final demands for the 3 products are $d_1 = 10$, $d_2 = 5$, and $d_3 = 6$.

- a. What are the primary inputs of each product?
- b. What is the required output of the 3 industries?
- c. Find the total required amount of primary inputs for this economy.