Lesson 15. More Economic Applications of Linear Systems

1 Overview

- In this lesson/worksheet, you will solve two types of economic models using the techniques for solving systems of linear equations we covered in Lessons 12 and 13.

2 Market models

- Consider the following **two commodity market model**:

$$D_1 = S_1 \quad (1) \qquad D_2 = S_2 \quad (4)$$
$$D_1 = 10 - 2P_1 + P_2 \quad (2) \qquad D_2 = 15 + P_1 - P_2 \quad (5)$$
$$S_1 = -2 + 3P_1 \quad (3) \qquad S_2 = -1 + 2P_2 \quad (6)$$

where

D_1 = demand for product 1 $\qquad$ D_2 = demand for product 2
S_1 = supply for product 1 $\qquad$ S_2 = supply for product 2
P_1 = price for product 1 $\qquad$ P_2 = price for product 2

- Look at equations (2) and (5). Explain why product 1 and product 2 are substitutes. (See Lesson 10 for a refresher.)

- We want to find the equilibrium prices P_1 and P_2 in this market.

- First, simplify the system (1)-(6) above. Because of equation (1), you can set the right hand sides of equations (2) and (3) equal to each other. You should obtain an equation with 2 variables: P_1 and P_2. Simplify the equation by collecting terms and putting all the P_1 and P_2 terms on the left, and the constant on the right.

(A)

- Do the same with equations (4), (5) and (6):

(B)

- Putting together the equations you found in (A) and (B), you should end up with the following system of equations:

$$
\begin{aligned}
-5P_1 + P_2 &= -12 \\
P_1 - 3P_2 &= -16
\end{aligned}
\tag{C}
$$

(Did you get the same equations? You may have the same equations, but multiplied by −1, which is OK.)

- Rewrite the system (C) in matrix form $AX = B$:

$A =$ $X =$ $B =$

- Now, use Cramer's rule to solve this system and find the equilibrium prices:

- You should find that the equilibrium prices are $P_1 = \frac{26}{7}$ and $P_2 = \frac{46}{7}$.

3 A model for national income

- Consider the following **national income model**:

$$Y = C + I_0 + G_0 \tag{7}$$

$$C = a + bY \qquad (0 < b < 1) \tag{8}$$

where

$$
\begin{aligned}
Y &= \text{national income} \\
C &= \text{consumer expenditure} \\
I_0 &= \text{business expenditure (i.e., investment)} \\
G_0 &= \text{government expenditure}
\end{aligned}
$$

(We saw a more complicated version of this model back in Lesson 7.)

- Equation (7) says that national income equals total expenditure by consumers, business, and government.

- What does equation (8) say about the relationship between consumer expenditure and total national income?

- Now suppose that $I_0 = 8$, $G_0 = 5$, $a = 4$, and $b = \frac{1}{3}$. Then equations (7) and (8) become

$$Y = C + 13 \tag{9}$$

$$C = 4 + \frac{1}{3}Y \tag{10}$$

- We want to solve for the national income Y and consumer expenditures C.
- First, simplify equations (9) and (10) by putting the Y and C terms on the left, and the constants on the right:

(D)

- Rewrite the system of equations you wrote in (D) in matrix form $AX = B$:

$A =$ $X =$ $B =$

- Now, use Cramer's rule to solve this system and find the national income and consumer expenditure:

- You should find that the national income $Y = \frac{51}{2}$ and consumer expenditure $C = \frac{25}{2}$.

4 Exercises

Problem 1. Consider the <u>three</u> commodity market model given by

$$\begin{aligned} D_1 &= S_1 & D_2 &= S_2 & D_3 &= S_3 \\ D_1 &= 2 - P_1 + 2P_2 + 2P_3 & D_2 &= 5 - 2P_1 - P_2 + 2P_3 & D_3 &= 5 - 2P_1 + 2P_2 - P_3 \\ S_1 &= -2 + P_1 & S_2 &= -1 + P_2 & S_3 &= -3 + P_3 \end{aligned}$$

Using a similar method to the one outlined in Section 2 of this worksheet, simplify the above model into a system of 3 linear equations and 3 variables P_1, P_2, and P_3. Solve this system to find the equilibrium prices P_1, P_2, and P_3 by forming the augmented matrix of the system and finding the RREF.

Problem 2. Suppose that in a national income model as in Section 3, we have $I_0 = 12$, $G_0 = 4$, $a = 1$, $b = \frac{1}{4}$. Use Cramer's rule to find Y and C.