Name: Notes

SM275 · Mathematical Methods for Economics Fall 2019 · Uhan

Quiz 3 – 11 September 2019

Instructions. You have 15 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Problem	Weight	Score
1	1	
2	1	
3	2	
4	2	
Total		/ 60

For Problems 1 and 2, consider the discrete market model

$$D_t = S_t$$
$$D_t = 22 - 3P_t$$
$$S_t = -2 + P_{t-1}$$

where at time t, D_t is the demand, S_t is the supply, and P_t is the price. In addition, suppose $P_0 = 8$. Using methods similar to those we used in class, we can rewrite this model as the following DS:

$$P_{t+1} = -\frac{1}{3}P_t + 8 \qquad t = 0, 1, 2, \ldots$$

The particular solution is

$$P_t = 2\left(-\frac{1}{3}\right)^t + 6.$$

Problem 1. Find the fixed point of the DS.

- Note that the setup of this problem is from Problem 9.7 in the textbook, assigned for homework.
- There are multiple ways to find the fixed point here:
  - the formula for the fixed point of a discrete market model DS in Lesson 5
  - the formula for the fixed point of a first-order linear DS in Lesson 5
  - the technique for finding a fixed point ofa first-order DS in Lesson 1

Problem 2. Is the fixed point attracting, repelling, or neither? Briefly explain.

- Take a look at Lesson 5 if you need a hint on how to get started.

For Problems 3 and 4, consider the DS

$$A_{n+2} = 5A_{n+1} - 6A_n + 8 \qquad n = 0, 1, 2, \ldots$$

Problem 3. Find the general solution.

- Note that Problems 3 and 4 come from Problem 10.1 in the textbook, assigned for homework.
- Be careful when identifying a, b, and c – especially with negative signs.
- Take a look at Lesson 6 if you need a hint on how to get started.

Problem 4. Find the particular solution satisfying the IC $A_0 = 1$, $A_1 = 2$.

Second order linear DS: $A_{n+2} = aA_{n+1} + bA_n + c$, $n = 0, 1, 2, \ldots$

- **Characteristic equation**: $x^2 = ax + b$ with roots r, s
- **General solution**:
  - If $a + b \neq 1$: $A_n = \begin{cases} c_1 r^n + c_2 s^n + \frac{c}{1-a-b} & \text{if } r \neq s \\ (c_1 + c_2 n) r^n + \frac{c}{1-a-b} & \text{if } r = s \end{cases}$ for any values of c_1, c_2
  - If $a + b = 1$: $A_n = \begin{cases} c_1(a-1)^n + c_2 + \left(\frac{c}{2-a}\right) n & \text{if } a + b = 1, a \neq 2 \\ c_1 + c_2 n + \left(\frac{c}{2}\right) n^2 & \text{if } a = 2, b = -1 \end{cases}$ for any values of c_1, c_2