Name: Notes

Quiz 4 – 18 September 2019

Problem	Weight	Score
1	1	
2	1	
3	1	
4	1	
5	1	
Total		/ 50

Instructions. You have 15 minutes to complete this quiz. You may use your calculator. You may not use any other materials (e.g., notes, homework, books).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Recall the national income model from Lesson 7, with marginal propensity to consume $m = \frac{1}{2}$ and accelerator $\ell = \frac{1}{6}$:

$$\begin{aligned} T_n &= C_n + I_n + G_n \\ C_{n+1} &= \frac{1}{2} T_n \\ I_{n+1} &= \frac{1}{6}(C_{n+1} - C_n) \\ G_n &= 1 \end{aligned} \qquad n = 0, 1, 2, \dots$$

where at time n, T_n is the total national income, C_n is the amount of consumer expenditures, I_n is the amount of private investment, and G_n is the amount of government expenditures. We showed that we can rewrite this model as the following DS:

$$T_{n+2} = \frac{7}{12} T_{n+1} - \frac{1}{12} T_n + 1 \qquad n = 0, 1, 2, \dots \qquad (\star)$$

Problem 1. Find the general solution to the DS ($\star$).

- Most of you answered this correctly.
- Be careful when identifying and plugging in a, b, c.

Problem 2. Suppose $C_0 = 4$ and $I_0 = 5$. Find the IC for the DS ($\star$).

- See Example 1c in Lesson 7 for a similar example.
- Remember the definition of the IC for a second order DS (Lesson 6).
- Some of you went ahead and found the particular solution that satisfies the IC. That was not necessary for this problem.

For Problems 3, 4 and 5, consider the following DS:

$$A_{n+2} = \frac{1}{2}A_{n+1} - \frac{3}{64}A_n + 35 \qquad n = 0, 1, 2, \ldots \tag{$\star\star$}$$

The general solution to the DS ($\star\star$) is

$$A_n = c_1\left(\frac{1}{8}\right)^n + c_2\left(\frac{3}{8}\right)^n + 64.$$

Problem 3. Show that the fixed point of the DS ($\star\star$) is 64.

- Most of you answered this correctly.
- Note that in general, the limit of A_n as $n \to \infty$ does not necessarily give you the fixed point of the DS.

Problem 4. Is the system stable, unstable, or neither? Briefly explain.

- Be careful and precise with your explanations. For example, the statement "the values of c_1 and c_2 approach 0 as n approaches ∞" doesn't make sense here – c_1 and c_2 are constants.
- Be careful with the free constants c_1 and c_2:
  - In order for the DS to be stable, A_n must approach a finite value as $n \to \infty$ for all initial conditions. This means $\lim_{n\to\infty} A_n$ must be finite no matter what c_1 and c_2 are.
  - In order for the DS to be unstable, $|A_n| \to \infty$ as $n \to \infty$ for some initial conditions. This means $\lim_{n\to\infty} A_n = \infty$ for at least one combination of values of c_1 and c_2.

Problem 5. Is the fixed point 64 attracting, repelling, or neither? Briefly explain.

- The notes for Problem 4 apply here as well.
- This is a second-order linear DS with $a + b \neq 1$. So, there is a unique fixed point.
  - To show the fixed point is attracting, you need to show that A_n approaches the fixed point as $n \to \infty$ for all initial conditions (Lesson 8).
  - To show the fixed point is repelling, you need to show that $|A_n| \to \infty$ as $n \to \infty$ for some initial conditions (Lesson 8).

The general solution of the **second order linear DS** $A_{n+2} = aA_{n+1} + bA_n + c$, $n = 0, 1, 2, \ldots$ is

$A_n = c_1 r^n + c_2 s^n + \dfrac{c}{1-a-b}$	if $a + b \neq 1$ and $r \neq s$
$A_n = (c_1 + c_2 n)r^n + \dfrac{c}{1-a-b}$	if $a + b \neq 1$ and $r = s$
$A_n = c_1(a-1)^n + c_2 + \dfrac{c}{2-a}n$	if $a + b = 1$ and $a \neq 2$
$A_n = c_1 + c_2 n + \dfrac{c}{2}n^2$	if $a + b = 1$ and $a = 2, b = -1$

where r and s are the roots of the characteristic equation $x^2 = ax + b$.