FORMULA TABLE

SM275 Math for Economics, Fall Semester 2019-2020

1. Continuously compounded interest: The final amount A in an account is given by:

$$A = A_0 e^{rt}$$

where A_0 is the intial amount, r is the interest rate and t is the number of years.

2. The general solution of the first order linear DS: $A_{n+1} = sA_n + b$ is

$$A_n = ks^n + \frac{b}{1-s}, \quad if \quad s \neq 1$$

and

$$A_n = k + nb, \quad if \quad s = 1$$

3. The general solution of the second order linear DS: $A_{n+2} = aA_{n+1} + bA_n + c$ is

$$A_n = C_1 r^n + C_2 s^n + \frac{c}{1-a-b} \quad if \quad a+b \neq 1 \quad and \quad r \neq s$$

$$A_n = (C_1 + C_2 n) r^n + \frac{c}{1-a-b} \quad if \quad a+b \neq 1 \quad and \quad r = s$$

$$A_n = C_1 (a-1)^n + C_2 + \frac{c}{2-a} n \quad if \quad a+b = 1 \quad and \quad a \neq 2$$

$$A_n = C_1 + C_2 n + \frac{c}{2} n^2 \qquad if \quad a+b = 1, a = 2 \quad and \quad b = -1$$

where r and s are the roots of the characteristic equation $x^2 = ax + b$.

4. A model for the National Economy is

$$T_{n+2} = m(1+l)T_{n+1} - mlT_n + 1,$$

where T = total national income, C = consumer expenditures, I = private investment, and G = government expenditures and

$$T_n = C_n + I_n + G_n$$

$$C_{n+1} = mT_n, m > 0$$

$$I_{n+1} = l(C_{n+1} - C_n), \quad l > 0$$

$$G_n = 1$$

The constant m is called the marginal propensity to consume (MPC) and the constant l is called the accelerator.

5. **The Second Derivative Test - General Case**: Suppose that P is a critical point of $f(x_1, \ldots, x_n)$. Recall that d_k is the determinant $|H_k|$, where H_k is the $k \times k$ submatrix in the upper left corner of the Hessian matrix of f at P.

   Assume first that $d_n \neq 0$. Then

   (a) If $d_k > 0$ for $k = 1, 2, \ldots, n$, then f has a local minimum at P.

   (b) If $d_k < 0$ for k odd and $d_k > 0$ for k even, then f has a local maximum at P.

   (c) If neither (a) nor (b) holds, then f has a saddle point at P.

   If $d_n = 0$, then the test provides no information.

6. **Second Derivative Test for Constrained Extrema**: Suppose $(\lambda, \mathbf{x_0})$ is a constrained critical point of $f(x_1, \ldots, x_n)$. Recall that d_j is the determinant $|H_j|$, where H_j is the $j \times j$ submatrix in the upper left corner of the bordered Hessian matrix $H_L(\lambda, \mathbf{x_0})$.

   Calculate the following sequence of numbers:

$$\{(-1)^k d_{2k+1}, (-1)^k d_{2k+2}, \ldots, (-1)^k d_{k+n}\}.$$

   Assume $d_{k+n} \neq 0$. Then

   (a) If the sequence consists entirely of positive numbers, then f has a constrained local minimum at $\mathbf{x_0}$.

   (b) If the sequence in alternates as negative, positive, negative, …, then f has a constrained local maximum at $\mathbf{x_0}$.

   (c) If neither case (a) nor case (b) holds, then f has a constrained saddle point at $\mathbf{x_0}$.

   If $d_{k+n} = 0$, then the test fails.