Name:

SM286A – Mathematics for Economics — Fall 2015

Asst. Prof. Nelson Uhan

Exam 1

Instructions

- You have 50 minutes to complete this exam.
- There are 7 problems on this exam, worth a total of 100 points.
- You may not consult any outside materials (e.g. notes, textbooks, homework).
- You may not use a calculator.
- **Show all your work.** Your answers should be legible and clearly labeled. It is your responsibility to make sure that I understand what you are doing. You will be awarded partial credit if your work merits it.
- Keep this booklet intact.
- **Do not discuss the contents of this exam with any midshipmen until the end of 4th period.**

Score

Problem	Maximum Points	Your Points
1	20	
2	10	
3	20	
4	10	
5	15	
6	15	
7	10	
Total	**100**	

Problem 1. (20 total points = 4 parts × 5 points)

Let

$$A = \begin{bmatrix} 1 & -2 \\ -3 & 0 \\ 2 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 2 \\ 7 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} 8 & -3 \\ 0 & 1 \end{bmatrix}$$

Compute the following quantities, or state that the quantity is not defined.

a. $2B + C$

b. CA

c. AC

d. A'

Problem 2. (10 total points)

Consider the following system of equations:

$$\begin{aligned} x \quad\quad + \; z &= 2 \\ 4y - 2z &= 8 \\ 4x - \; y + 5z &= 4 \end{aligned}$$

You are given that

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & -2 \\ 4 & -1 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 9 & -1/2 & -2 \\ -4 & 1/2 & 1 \\ -8 & 1/2 & 2 \end{bmatrix}$$

Use this information to solve for x, y, and z.

Problem 3. (20 total points)

Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 4 & 2 \end{bmatrix}$.

a. (15 points) Find the inverse of A. (You may assume it exists.)

b. (5 points) Does $|A| = 0$? Why?

Problem 4. (10 total points)

Suppose you have a system of 4 linear equations with 5 variables: x_1, x_2, x_3, x_4, and x_5. You form the augmented matrix of this system, and find its RREF:

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

What are the solutions to the original system of equations? If there are no solutions, simply state so. If there is at least one solution, write your answer in vector form. Your answer should look like this:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

Problem 5. (15 total points)

a. (7 points) Find the determinant of $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 2 & 5 & 4 \\ 0 & -1 & 0 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix}$.

b. (8 points) Use Cramer's rule to solve the following system of equations for x and y:

$$\begin{aligned} -2x + y &= a \\ x + 3y &= b \end{aligned}$$

Problem 6. (15 total points)

Recall the two-commodity market equilibrium model we discussed in class:

$$
\begin{aligned}
Q_{d1} &= Q_{s1} \\
Q_{d1} &= a_0 + a_1 P_1 + a_2 P_2 \\
Q_{s1} &= b_0 + b_1 P_1 + b_2 P_2 \\
Q_{d2} &= Q_{s2} \\
Q_{d2} &= \alpha_0 + \alpha_1 P_1 + \alpha_2 P_2 \\
Q_{s2} &= \beta_0 + \beta_1 P_1 + \beta_2 P_2
\end{aligned}
$$

Variables: Q_{d1} = quantity demanded for product 1

Q_{d2} = quantity demanded for product 2

Q_{s1} = quantity supplied for product 1

Q_{s2} = quantity supplied for product 2

P_1 = price of product 1

P_2 = price of product 2

Parameters: $a_0, a_1, a_2, b_0, b_1, b_2, \alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2$

a. (10 points) Rewrite the model above into matrix form, assuming the six variables are arranged in the order Q_{d1}, Q_{d2}, Q_{s1}, Q_{s2}, P_1, and P_2. Your answer should look like this:

$$
\begin{bmatrix} \\ \\ \text{some} \\ \text{matrix} \\ \\ \\ \end{bmatrix}
\begin{bmatrix} Q_{d1} \\ Q_{d2} \\ Q_{s1} \\ Q_{s2} \\ P_1 \\ P_2 \end{bmatrix}
=
\begin{bmatrix} \\ \\ \text{some other} \\ \text{matrix} \\ \\ \\ \end{bmatrix}
$$

b. (5 points) Suppose $a_2 < 0$ and $\alpha_1 < 0$. What is the relationship between the two products? Why?

Problem 7. (10 total points)

Consider a Leontief input-output model for an economy with 2 industries, with input matrix A and final demand vector d:

$$A = \begin{bmatrix} 0.38 & 0.12 \\ 0.19 & 0 \end{bmatrix} \qquad d = \begin{bmatrix} 250 \\ 600 \end{bmatrix}$$

a. (5 points) What is the economic meaning of 0.19 in the input matrix A above?

b. (5 points) Let

$$x_1 = \text{output of industry 1, in dollars}$$
$$x_2 = \text{output of industry 2, in dollars}$$

Write the input-output matrix equation for this model — i.e., the matrix equation that ensures that each industry's output is equal to the input demand and the final demand for its product. Your answer should look like this:

$$\begin{bmatrix} \text{some} \\ \text{matrix} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \text{some other} \\ \text{matrix} \end{bmatrix}$$