SM286A – Mathematics for Economics Fall 2015
Asst. Prof. Nelson Uhan

# Exam 2 – Information

## 1 Exam format

- When: **Thursday 29 October** in class
- How long: 50 minutes (1 period)
- What: Lessons 10-19
- One 3 in × 5 in index card of handwritten notes (both sides) allowed
- No other outside materials allowed
- No calculators

## 2 Schedule

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|---|---|
| **Tuesday 27 October** | Double period class: Review |
| **Wednesday 28 October** | Exam EI, 19:00-20:30, CH348 |
| **Thursday 29 October** | Single period class: Exam |

## 3 Review Problems

This collection of problems is not meant to represent the length of the exam. You are responsible for all the material covered in Lessons 10-19, not just what is represented in the problems below.

**Problem 1.** Solve the differential equation $\frac{dy}{dt} - 2ty = t$ with the initial condition $y(0) = 1$. Verify your solution.

**Problem 2.** Is the differential equation $(1 + t^2)\,dy + 2ty\,dt = 0$ is exact? Why? Solve this differential equation.

**Problem 3.** Is the differential equation $\frac{1}{y}\,dy + \frac{2t}{1+t^2}\,dt = 0$ separable? Why? Solve this differential equation.

**Problem 4.** Solve the differential equation $\frac{dy}{dt} = \frac{-2ty}{1+t^2}$.

**Problem 5.** Solve the differential equation $\frac{dy}{dt} - \frac{3}{t}y = t^4 y^{1/3}$.

**Problem 6.** Plot the phase line for the differential equation $\frac{dy}{dt} = y^2 - 4y + 3$. What are the equilibrium points? Are they dynamically stable or unstable?

**Problem 7.** Solve the difference equation $y_{t+1} - \frac{1}{2}y_t = 3$ with the initial condition $y_0 = 5$. Verify your solution. Is $y_t$ oscillating? Is $y_t$ divergent?

**Problem 8.** Consider the following model of a market with a single product in continuous time. The model variables are

$P$ = unit price $\quad$ $Q_d$ = quantity demanded $\quad$ $Q_s$ = quantity supplied

The model equations are:

$$\frac{dP}{dt} = \frac{1}{2}(Q_d - Q_s)$$
$$Q_d = 5 - 2P + \frac{dP}{dt}$$
$$Q_s = -1 + 3P$$

a. Combine these equations to find a single equivalent differential equation.
b. Solve this differential equation.
c. What can you say about the price of the product in the long run?

**Problem 9.** Consider the following version of the Solow growth model. The model variable is

$k$ = capital-to-labor ratio

The model equation is

$$\dot{k} = 2k^{1/2} - k$$

a. Plot the phase line of the model equation.
b. What are the equilibrium points? Are they dynamically stable or unstable?
c. What can you say about the capital-to-labor ratio in the long run?

**Problem 10.** Consider the following model of a market with a single product in discrete time. The model variables are

$P_t$ = unit price in period $t$ $\quad$ $Q_{dt}$ = quantity demanded in period $t$ $\quad$ $Q_{st}$ = quantity supplied in period $t$

The model equations are

$$Q_{dt} = Q_{st}$$
$$Q_{dt} = 5 - 2P_t$$
$$Q_{st} = -1 + 4P_{t-1}$$

a. Combine these equations to find a single equivalent difference equation.
b. Solve this difference equation.
c. What can you say about the price of the product in the long run?