SM286A – Mathematics for Economics Fall 2015
Asst. Prof. Nelson Uhan

Exam 3 – Information

1 Exam format

- When: **Thursday 3 December** in class
- How long: 50 minutes (1 period)
- What: Lessons 20-26
- One 3 in × 5 in index card of handwritten notes (both sides) allowed
- You may use your calculator
- No other outside materials allowed

2 Schedule

Tuesday 1 December	Double period class: Review and EI
Wednesday 2 December	EI, 19:00-20:30, CH348
Thursday 3 December	Single period class: Exam

3 Review Problems

This collection of problems is not meant to represent the length of the exam. You are responsible for all the material covered in Lessons 20-26, not just what is represented in the problems below.

Problem 1. Find the local optima of $f(x_1, x_2) = 8x_1^3 - 12x_1x_2 + x_2^3$.

Problem 2. Find the local optima of $f(x_1, x_2, x_3) = -x_1^4 - 2x_2^2 - x_3^2 + 4x_1x_2 + 2x_3$.

Problem 3. Is $f(x_1, x_2, x_3) = x_1^2 + 4x_2^2 + 2x_3^2 + x_1x_3 + x_2x_3 + 8$ strictly convex or strictly concave? Why? Suppose $f(0,0,0) = 8$ is a local minimum of f. Is $f(0,0,0) = 8$ also an absolute minimum? Why?

Problem 4. Find the local optima of $f(x_1, x_2) = -x_1^2 + x_2^2$ subject to the constraint $x_1^2 + 4x_2^2 = 4$.

Problem 5. Find the local optima of $f(x_1, x_2, x_3) = x_1x_2x_3$ subject to the constraint $x_1 + 2x_2 + 3x_3 = 6$.

Problem 6. Find the local optima of $f(x_1, x_2, x_3) = x_1$ subject to the constraints $x_1 - x_2^2 - 2x_3^2 = 0$ and $x_2^2 + x_3^2 = 1$.

Problem 7. Consider a firm that produces and sells two products. Below is a model that represents the firm's profit maximization problem.

- Variables:

Q_1 = quantity of product 1 produced and sold P_1 = unit price of product 1
Q_2 = quantity of product 2 produced and sold P_2 = unit price of product 2

- Model:

$$\begin{aligned} \text{maximize} \quad & P_1Q_1 + P_2Q_2 - 12 - 4Q_1 - 8Q_2 \\ \text{subject to} \quad & P_1 = 46 - 3Q_1 \\ & P_2 = 32 - 2Q_2 \end{aligned}$$

a. Describe the objective function and the constraints of the model. (e.g. What are the prices of the products? What are the costs of production? Do the prices depend on demand?)
b. By substituting the constraints into the objective function, find the local maximum values of profit.
c. Using the Lagrange multiplier method, find the local maximum values of profit.
d. What do your answers from parts b and c tell you about what the firm should do?

Problem 8. Consider a firm that produces a good that requires two inputs to produce. In particular, x_1 units of input 1 and x_2 units of input 2 yield $3x_1^{1/3}x_2^{1/3}$ units of the good. Each unit of input 1 costs \$20, and each unit of input 2 costs \$160.

a. Using the variables x_1 and x_2 defined above, write cost as a function of x_1 and x_2: $c(x_1, x_2) = \ldots$
b. The firm needs to produce 24 units of the product. Using the variables x_1 and x_2 defined above, write an equality constraint that models this.
c. Using the Lagrange multiplier method, find the local minimum values of cost.
d. What does your answer from part c tell you about what the firm should do?