Example 6. Solve for the equilibrium prices of this two-commodity market equilibrium model by finding the RREF of the augmented matrix of the above system.

$$\begin{bmatrix} 5 & -1 & 84 \\ 1 & -3 & -112 \end{bmatrix} \xrightarrow{\text{Swap } R_1, R_2} \begin{bmatrix} 1 & -3 & -112 \\ 5 & -1 & 84 \end{bmatrix} \xrightarrow{R_2 - 5(R_1)} \begin{bmatrix} 1 & -3 & -112 \\ 0 & 14 & 644 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{14}R_2} \begin{bmatrix} 1 & -3 & -112 \\ 0 & 1 & 46 \end{bmatrix} \xrightarrow{R_1 + 3(R_2)} \begin{bmatrix} 1 & 0 & 26 \\ 0 & 1 & 46 \end{bmatrix}$$

Solution: $P_1 = 26$, $P_2 = 46$

Example 7. Solve for the equilibrium prices of this two-commodity market equilibrium model by finding the inverse of the coefficient matrix of the above system.

$$\begin{bmatrix} 5 & -1 & 1 & 0 \\ 1 & -3 & 0 & 1 \end{bmatrix} \xrightarrow{\text{swap } R_1, R_2} \begin{bmatrix} 1 & -3 & 0 & 1 \\ 5 & -1 & 1 & 0 \end{bmatrix} \xrightarrow{R_2 - 5(R_1)} \begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 14 & 1 & -5 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{14}(R_2)} \begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 1 & \frac{1}{14} & -\frac{5}{14} \end{bmatrix} \xrightarrow{R_1 + 3(R_2)} \begin{bmatrix} 1 & 0 & \frac{3}{14} & -\frac{1}{14} \\ 0 & 1 & \frac{1}{14} & -\frac{5}{14} \end{bmatrix}$$

$\Rightarrow$ The inverse of the coefficient matrix is $\begin{bmatrix} \frac{3}{14} & -\frac{1}{14} \\ \frac{1}{14} & -\frac{5}{14} \end{bmatrix}$

Matrix form of system:

$$\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 84 \\ -112 \end{bmatrix} \iff \underbrace{\begin{bmatrix} \frac{3}{14} & -\frac{1}{14} \\ \frac{1}{14} & -\frac{5}{14} \end{bmatrix}}_{= I_2} \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} \frac{3}{14} & -\frac{1}{14} \\ \frac{1}{14} & -\frac{5}{14} \end{bmatrix} \begin{bmatrix} 84 \\ -112 \end{bmatrix}$$

$$\iff \text{Solution: } \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} \frac{3}{14} & -\frac{1}{14} \\ \frac{1}{14} & -\frac{5}{14} \end{bmatrix} \begin{bmatrix} 84 \\ -112 \end{bmatrix} = \begin{bmatrix} 26 \\ 46 \end{bmatrix}$$

5 If we have time...

Example 8 (Also a homework problem). What is the rank of $\begin{bmatrix} 7 & 6 & 3 & 3 \\ 0 & 1 & 2 & 1 \\ 8 & 0 & 0 & 8 \end{bmatrix}$? Is this matrix nonsingular?