Lesson 8. Cramer's Rule, Applications to Economic Models

0 Warm up

Example 1. Find the following determinants:

a. $\begin{vmatrix} 2 & 3 & 0 \\ 0 & 4 & 5 \\ 6 & 0 & 7 \end{vmatrix}$ b. $\begin{vmatrix} 8 & 3 & 0 \\ 3 & 4 & 5 \\ -1 & 0 & 7 \end{vmatrix}$ c. $\begin{vmatrix} 2 & 8 & 0 \\ 0 & 3 & 5 \\ 6 & -1 & 7 \end{vmatrix}$ d. $\begin{vmatrix} 2 & 3 & 8 \\ 0 & 4 & 3 \\ 6 & 0 & -1 \end{vmatrix}$

1 Cramer's rule

- Suppose we want to solve a system of equations $Ax = d$ for x, where A is $n \times n$ and d is $n \times 1$
  - Quick check: x has dimension
- Let A_j be the matrix A, but with the jth column replaced by d
- **Cramer's rule:**

Example 2. Solve the following system of equations using Cramer's rule:

$$\begin{aligned} 2x_1 + 3x_2 \qquad &= 8 \\ 4x_2 + 5x_3 &= 3 \\ 6x_1 \qquad + 7x_3 &= -1 \end{aligned}$$

2 Two commodity partial market equilibrium

- Market with two products that are related to each other
- Variables:

$$Q_{d1} = \text{quantity demanded for product 1} \qquad Q_{d2} = \text{quantity demanded for product 2}$$
$$Q_{s1} = \text{quantity supplied for product 1} \qquad Q_{s2} = \text{quantity supplied for product 2}$$
$$P_1 = \text{price of product 1} \qquad P_2 = \text{price of product 2}$$

- A general model with 6 variables and 6 equations:

$$\begin{aligned} Q_{d1} &= Q_{s1} & Q_{d2} &= Q_{s2} \\ Q_{d1} &= a_0 + a_1P_1 + a_2P_2 & Q_{d2} &= \alpha_0 + \alpha_1P_1 + \alpha_2P_2 \\ Q_{s1} &= b_0 + b_1P_1 + b_2P_2 & Q_{s2} &= \beta_0 + \beta_1P_1 + \beta_2P_2 \end{aligned}$$

- Depending on the economic context, the parameters $a_0, a_1, a_2, b_0, b_1, b_2, \alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2$ will have particular signs, magnitudes or relationships between each other
  - Product 1 and product 2 are **substitutes** if:
  - Product 1 and product 2 are **complements** if:
- Using the equilibrium conditions, we can simplify the above model into 2 variables and 2 equations:

$$\begin{aligned} (a_1 - b_1)P_1 + (a_2 - b_2)P_2 &= -(a_0 - b_0) \\ (\alpha_1 - \beta_1)P_1 + (\alpha_2 - \beta_2)P_2 &= -(\alpha_0 - \beta_0) \end{aligned} \quad \Leftrightarrow$$

- Using Cramer's rule, we can find the equilibrium prices:
- Using this closed form solution, we can analytically determine the effects of the parameters on the equilibrium prices

3 A national income model

- Variables:

  Y = national income $\qquad$ C = (planned) consumption expenditure

- Parameters:

  I_0 = investment expenditures $\qquad$ G_0 = government expenditures

  a = autonomous consumption expenditure $\qquad$ b = marginal propensity to consume

- Model:

$$\begin{aligned} Y &= C + I_0 + G_0 \\ C &= a + bY \end{aligned} \qquad (a > 0, 0 < b < 1)$$

- How is consumption related to national income in this model?

Example 3.

a. Rewrite the national income model above in matrix form, listing the variables in the order Y, C.
b. Solve for variables Y and C using Cramer's rule.