4 The existence of nonnegative solutions

- Leontief matrix $I - A$ is nonsingular $\Rightarrow$ solutions to the model exist
- Ideally, the solutions to the model would also be nonnegative
- When does this happen?
- The Leontief matrix $I - A$ has the form:

$$\begin{bmatrix} 1-a_{11} & -a_{12} & -a_{13} & \cdots & -a_{1n} \\ -a_{21} & 1-a_{22} & -a_{23} & \cdots & -a_{2n} \\ -a_{31} & -a_{32} & 1-a_{33} & \cdots & -a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & -a_{n3} & \cdots & 1-a_{nn} \end{bmatrix} \leftarrow$$

use the n square submatrices of $I-A$ starting at the 1,1 entry (top left corner)

- The **leading principal minors** of $I - A$ are:

$$|1-a_{11}|, \quad \begin{vmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{vmatrix}, \quad \begin{vmatrix} 1-a_{11} & -a_{12} & -a_{13} \\ -a_{21} & 1-a_{22} & -a_{23} \\ -a_{31} & -a_{32} & 1-a_{33} \end{vmatrix}, \ldots, \begin{vmatrix} 1-a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & 1-a_{22} & \cdots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & \cdots & 1-a_{nn} \end{vmatrix}$$

total of n leading principal minors

- **Hawkins-Simon condition.** The system $(I - A)x = d$ has a nonnegative solution if and only if the leading principal minors of $I - A$ are all positive

Example 3. Verify the Hawkins-Simon condition holds in Example 2.

$$|0.8| = 0.8 > 0 \checkmark$$

$$\begin{vmatrix} 0.8 & -0.3 \\ -0.4 & 0.9 \end{vmatrix} = 0.84 > 0 \checkmark$$

$$\begin{vmatrix} 0.8 & -0.3 & -0.2 \\ -0.4 & 0.9 & -0.2 \\ -0.1 & -0.3 & 0.8 \end{vmatrix} = 0.384 > 0 \checkmark$$