

## Lesson 11. Dynamics of Market Price

### 1 Last time: a market equilibrium model with price dynamics

- Market with single commodity
- Variables:

$Q_d$  = quantity demanded       $Q_s$  = quantity supplied       $P$  = unit price

- Equations:

$$Q_d = \alpha - \beta P \quad (\alpha, \beta > 0) \quad (1)$$

$$Q_s = -\gamma + \delta P \quad (\gamma, \delta > 0) \quad (2)$$

$$\frac{dP}{dt} = j(Q_d - Q_s) \quad (j > 0) \quad (3)$$

- What does (3) say about how the price changes over time?
  - $Q_d > Q_s \Rightarrow dP/dt > 0 \Rightarrow$  price increases over time
  - $Q_d < Q_s \Rightarrow dP/dt < 0 \Rightarrow$  price decreases over time
  - $Q_d = Q_s \Rightarrow dP/dt = 0 \Rightarrow$  price stays the same over time
- Two types of equilibrium price:
  - **market-clearing equilibrium price:** demand equals supply, or  $Q_d = Q_s$
  - **intertemporal equilibrium price:** price is constant over time, or  $dP/dt = 0$
  - In this model, by (3), these two types of equilibrium price are the same
  - This may not be true in every model
- Substitute (1) and (2) into (3) to obtain the differential equation:

- This is a differential equation of the form

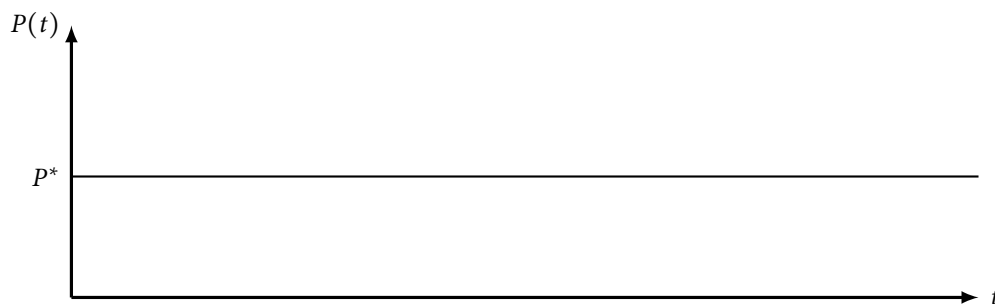
$$\frac{dy}{dt} + ay = b \quad \text{with definite solution} \quad y(t) = \left( y(0) - \frac{b}{a} \right) e^{-at} + \frac{b}{a}$$

- Therefore, we can find  $P(t)$ :

- In Lesson 1, we used (1) and (2) with the equilibrium condition  $Q_d = Q_s$  to find  $P^* = \frac{\alpha + \gamma}{\beta + \delta}$
- So, we can rewrite  $P(t)$  as

## 2 Dynamic stability

- As  $t \rightarrow \infty$ , the price  $P(t) \rightarrow$
- The price adjustment process defined by the model (1)-(3) is **dynamically stable**:  $P(t)$  converges to a constant  $P^*$  with enough time
- How does  $P(t)$  converge to  $P^*$ ?



## 3 Ensuring dynamic stability

- Suppose we do not assume  $\alpha, \beta, \gamma, \delta > 0$
- What restrictions do we need to impose on  $\alpha, \beta, \gamma, \delta$  to ensure dynamic stability?
- In order for  $P(t) \rightarrow P^*$ , we need

- In words:

- When the demand curve is negatively sloped (  ) and the supply curve is positively sloped (  ), this happens automatically