Lesson 12. Variable Coefficients

1 Overview

- General first-order linear differential equation:

$$\frac{dy}{dt} + u(t)y = w(t)$$

- Past lesson: solve for $y(t)$ when $u(t) = a$ and $w(t) = b$ are constants
- What happens when $u(t)$ and $w(t)$ are **variable** – that is, they depend on t?

2 The homogeneous case

- Suppose $w(t) = 0$, i.e. $\frac{dy}{dt} + u(t)y = 0$
- We can still use basic integration techniques to solve for $y(t)$:

- The general solution is
- We can get the definite solution by using an initial condition and solving for A

Example 1. Solve the equation $\frac{dy}{dt} + 3t^2 y = 0$ with the initial condition $y(0) = 4$.

3 The nonhomogeneous case

- When $w(t) \neq 0$, we need to work harder to obtain the solution
- We'll discuss this in the next lesson — in the meantime, here's the formula
- The general solution is

- Again, we can get the definite solution by using an initial condition and solving for A

Example 2. Solve the equation $\frac{dy}{dt} + 6y = -e^t$ with the initial condition $y(0) = \frac{6}{7}$.

4 Practice!

Example 3. Solve the equation $\frac{dy}{dt} + 2ty = t$ with the initial condition $y(0) = 1$.

Example 4. Solve the equation $\frac{dy}{dt} + y = t$ with the initial condition $y(0) = 2$.