- So there is an equilibrium $k = \bar{k}$ that is $\boxed{\text{dynamically stable}}$
- What does this mean economically?
  - At $k = \bar{k}$, we have $\dot{k} = 0$, or in words:

    capital-to-labor ratio is constant

  - Therefore, in the long run, capital must grow at the same rate as labor
  - Recall that we defined $\phi(k) = Q/L$. So, at $k = \bar{k}$, we have also that

$$Q = L\phi(\bar{k})$$

  - Therefore, in the long run, output must also grow at the same rate as labor

5 Quantitative analysis

Example 2. Suppose the production function in the Solow growth model is $f(K, L) = K^{3/4}L^{1/4}$.

a. Show that f is homogeneous: $f(aK, aL) = af(K, L)$.
b. Find $\phi(k)$.
c. Write the differential equation (S). Solve the equation for k. *Hint.* It is a Bernoulli equation!
d. What does k converge to as $t \to \infty$?

a. $f(aK, aL) = (aK)^{3/4}(aL)^{1/4} = a^{3/4}K^{3/4}a^{1/4}L^{1/4} = aK^{3/4}L^{1/4}$
$= af(K,L)$ ✓

b. $\phi(k) = \dfrac{K^{3/4}L^{1/4}}{L} = \left(\dfrac{K}{L}\right)^{3/4} = k^{3/4}$

c. $\dot{k} = s\phi(k) - \lambda k \Rightarrow \dot{k} = sk^{3/4} - \lambda k \Rightarrow \underline{\dot{k} + \lambda k = sk^{3/4}}$

Bernoulli: $R = \lambda$ $T = s$ $m = 3/4$

Let $z = k^{\frac{1}{4}}$ $\Rightarrow \dfrac{dz}{dt} + \underbrace{\tfrac{1}{4}\lambda}_{a} z = \underbrace{\tfrac{1}{4}s}_{b}$ ← 1st order linear DE w/constant coeffs – see Lesson 10

$\Rightarrow z(t) = Ae^{-\frac{1}{4}\lambda t} + \dfrac{s}{\lambda}$

$\Rightarrow k(t) = z(t)^4 = \left(Ae^{-\frac{1}{4}\lambda t} + \dfrac{s}{\lambda}\right)^4$

d. as $t \to \infty$, $k(t) \to \left(\dfrac{s}{\lambda}\right)^4$