Lesson 18. Dynamic Stability

1 Last time...

$$y_{t+1} + ay_t = c \qquad \text{has solution} \qquad y_t = \begin{cases} y_0 + ct & \text{if } a = -1 \\ \left(y_0 - \dfrac{c}{1+a}\right)(-a)^t + \dfrac{c}{1+a} & \text{if } a \neq -1 \end{cases}$$

- Notation: for this lesson, let's rewrite the above solution as

$$y_t = \begin{cases} A + ct & \text{if } b = 1 \ (a = -1) \\ Ab^t + \dfrac{c}{1-b} & \text{if } b \neq 1 \ (a \neq -1) \end{cases} \qquad \text{where } b = -a \text{ and } A \text{ is some constant}$$

2 Long-run behavior and the significance of b

- Does the solution y_t converge or diverge as $t \to \infty$? How does it converge or diverge?
- To investigate, let's consider what happens when $c = 0$
- In this case, the solution is

$$y_t = \begin{cases} \boxed{} & \text{if } b = 1 \\ \boxed{} & \text{if } b \neq 1 \end{cases}$$

- Depending on the value of b, y_t behaves differently as $t \to \infty$

Example 1. Assume $A = 1$. Determine y_0, y_1, y_2, y_3, y_4 for the different values of b given below.

b	y_0	y_1	y_2	y_3	y_4
2					
1					
1/2					
0					
-1/2					
-1					
-2					

- From Example 1, we can determine some trends
- When $b < 0$, y_t is
- When $b > 0$, y_t is
- When $|b| < 1$, y_t is
- When $|b| > 1$, y_t is
- These trends still hold for different values of A and c

Example 2. Describe the time path given by $y_t = 5(-1/10)^t + 3$.

Example 3. Describe the time path described by the difference equation $y_{t+1} + 2y_t = 9$.

3 The role of A

- If the magnitude of A changes:
- If the sign of A changes: