Lesson 19. The Cobweb Model

0 Warm up

Example 1. Solve the difference equation $y_{t+1} + 3y_t = 4$. Is y_t oscillatory or nonoscillatory? Is y_t convergent or divergent?

1 Overview

- Market with single product
- Supply in period t is decided in period $t-1$ and so is based on the price in period $t-1$
  - e.g. agricultural production: planting must occur a long time before harvesting and selling
- How does the price of the product change over time?

2 The model

- Variables:

$$P_t = \text{unit price at period } t$$
$$Q_{dt} = \text{quantity demanded at period } t$$
$$Q_{st} = \text{quantity supplied at period } t$$

- Parameters: $\alpha, \beta, \gamma, \delta > 0$
- Equations:

$Q_{dt} = Q_{st}$

$Q_{dt} = \alpha - \beta P_t$

$Q_{st} = -\gamma + \delta P_{t-1}$

3 Solving and analyzing the model

- Substituting the last two equations into the first, we can reduce the model to:

- Let's rewrite this difference equation by shifting the time subscripts:

- Note that in this case:

- We can solve for P_t, where P_0 represents the initial price:

- We can interpret $\bar{P} = \dfrac{\alpha + \gamma}{\beta + \delta}$ as the intertemporal equilbrium price of the model, and write P_t as:

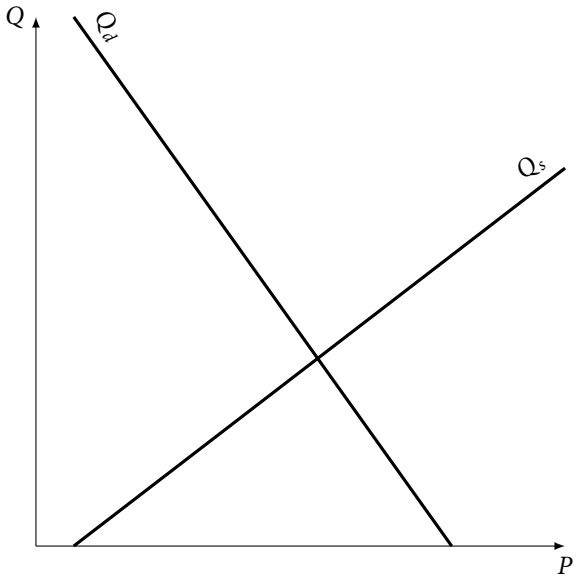
- P_t is convergent when

- P_t is oscillatory when

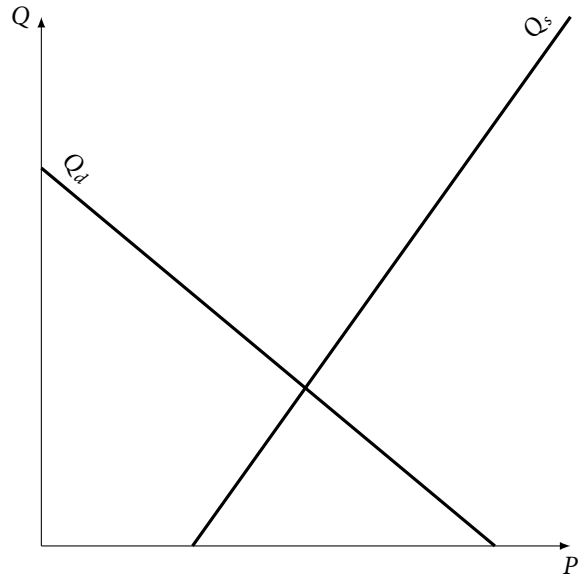
4 An alternative analysis: drawing cobwebs

- Sequence of events:
  - Given an initial price P_0, producers determine the supply in period 1
  - Market-clearing condition: supply in period 1 = demand in period 1
  - Given demand in period 1, determine price in period 1 that clears the market
  - Repeat!

Case 1: $\delta < \beta$
(supply curve flatter than demand curve)



Case 2: $\delta > \beta$
(supply curve steeper than demand curve)



- $\delta > \beta$: ____ oscillation
- $\delta = \beta$: ____ oscillation
- $\delta < \beta$: ____ oscillation

5 Economic insights

- Depending on the relationship between the slopes of the demand and supply curves, prices converge or diverge
- Prices can be subject to periodic fluctuations in these kinds of markets