Lesson 23. Profit Maximization

1 Incorporating demand into profit maximization

- Consider a firm that produces and sells three products
- The demand for these products depends on the prices of the products
- Variables:

R = revenue	Q_1 = quantity of product 1 produced and sold	P_1 = unit price of product 1
C = cost	Q_2 = quantity of product 2 produced and sold	P_2 = unit price of product 2

- Model:

$$
\begin{array}{ll}
\text{maximize} & R - C \\
\text{subject to} & R = P_1Q_1 + P_2Q_2 + P_3Q_3 \\
& C = 20 + 15(Q_1 + Q_2 + Q_3) \\
& P_1 = 63 - 4Q_1 \\
& P_2 = 105 - 5Q_2 \\
& P_3 = 75 - 6Q_3
\end{array}
$$

- Let's determine what the firm needs to produce and sell in order to maximize profit
- First, let's simplify the model
- We can express R as a function of Q_1, Q_2, Q_3 by substitution:

- Next, we can express profit π as a function of Q_1, Q_2, Q_3 by substitution as well:

- Now, let's maximize π

Step 1. Find the critical points

- The gradient of π is

- The first-order necessary condition tells us that critical points of π must satisfy

- Therefore, we have one critical point of π:

Step 2. Classify each critical point as a local minimum, local maximum, or saddle point

- The Hessian matrix of π is

- The Hessian matrix of π at the critical point $(Q_1, Q_2, Q_3) = (6, 9, 5)$ is

- The leading principal minors of the Hessian at $(Q_1, Q_2, Q_3) = (6, 9, 5)$ are

- Therefore, the second-order sufficient condition tells us that

Step 3. Is the function strictly concave or convex?

- The leading principal minors of the Hessian for any possible values of (Q_1, Q_2, Q_3) are

- Therefore, π is

- In addition, it follows that $\pi(6, 9, 5) = 679$ is

2 Determining amounts of capital and labor to maximize profit

- Suppose that a firm's production is a function of capital and labor
- Variables:

R = revenue	Q = quantity produced
C = cost	K = quantity of capital input
	L = quantity of labor input

- Model:

$$\begin{array}{rll} \text{maximize} & R - C & \\ \text{subject to} & R = 9Q & \\ & C = 3K + 3L & \\ & Q = K^{1/3}L^{1/3} & \text{(Cobb-Douglas production function)} \\ & K, L > 0 & \text{(restrict capital and labor to positive values)} \end{array}$$

- Let's determine the capital and labor needed to maximize profit
- First, let's simplify the model – by substitution, we can write profit as a function of capital and labor:

Step 1. Find the critical points

- The gradient of π is:

- The first-order necessary condition tells us that critical points of π must satisfy:

- Therefore, we have one critical point of π:

Step 2. Classify each critical point as a local minimum, local maximum, or saddle point

- The Hessian matrix of π is
- The Hessian matrix of π at the critical point $(K, L) = (1, 1)$ is
- The leading principal minors of the Hessian at $(K, L) = (1, 1)$ are
- Therefore, the second-order sufficient condition tells us that

Step 3. Is the function strictly concave or convex?

- The leading principal minors of the Hessian for any possible values of (K, L) are
- Therefore, π is
- In addition, it follows that $\pi(1, 1) = 3$ is